

Precalculus Summer Assignment

Instructions and Timeline

American School Foundation of Guadalajara

Teacher: Dr. June Mitsuhashi

Welcome to Precalculus!

This summer assignment is designed to help you review and reinforce key concepts from Integrated Math II, ensuring a strong foundation as we begin our journey into Precalculus. Completing this work independently and thoughtfully will give you the confidence and preparation needed to start the year successfully.

Assignment Overview

You are expected to complete the **Precalculus Summer Assignment** (linked in Google Docs) which reviews:

- Linear and quadratic functions
- Exponents and radicals
- Rational expressions
- Systems of equations
- Graphing basics

This assignment is to be **completed by hand** on paper and **submitted on the first day of school, Monday, August 19, 2025**.

Partial Deadlines and Suggested Timeline

While the full assignment is due in August, we strongly recommend following these checkpoints to stay on track:

Week	Dates	Focus Topics	Suggested Completion
1	June 23–29	Linear functions, slope, intercepts	Pages 1–2
2	June 30–July 6	Quadratic functions and factoring	Pages 3–4
3	July 7–13	Exponents and radicals	Pages 5–6
4	July 14–20	Rational expressions and equations	Pages 7–8
5	July 21–27	Systems of equations	Pages 9–10
6	July 28–August 3	Word problems and mixed review	Pages 11–12
7	August 4–10	Self-check, fix errors, finalize	Full review
8	August 11–18	Final polish & readiness check & Science Addendum	Ready to submit!

 **Tip:** Plan for about 1 hour per topic per week. Spread it out to avoid cramming!

 **NEW this year:** *Once you finish the Pre-Calculus Summer work, proceed to the Science Department Addendum and complete the exercises ONLY of your elective Science Course (note that AP science courses are not included in this addendum, as these have their separate summer assignment)*

Submission Requirements

- Bring your **completed handwritten work** on the first day of class (August 19).
- Your assignment will count as your **first graded formative** for Precalculus.
- Neatness, completeness, and effort **all matter**.

Looking forward to an exciting and empowering year ahead.
Enjoy your summer and keep your math brain active!

– Dr. Mitsuhashi

Precalculus/ Trigonometry Summer Assignment

Name: _____

Welcome to Precalculus/Trigonometry! Going into Precalculus/Trigonometry we will be building off prior concepts that you learned about in previous math courses. Sometimes this can be difficult if you have not reviewed these topics and are proficient in them. These are the topics that you need to review from your previous math course to be successful in Precalculus / Trigonometry . For your summer assignment, you will complete the following pages to review these topics.

Linear Functions (CP Only)

D Level

1. $7x - 22 < 11x + 45$
2. $2|x - 8| = 5$
3. If you graphed the function $y = |x - 1| - 4$, what would be the coordinates of the vertex?

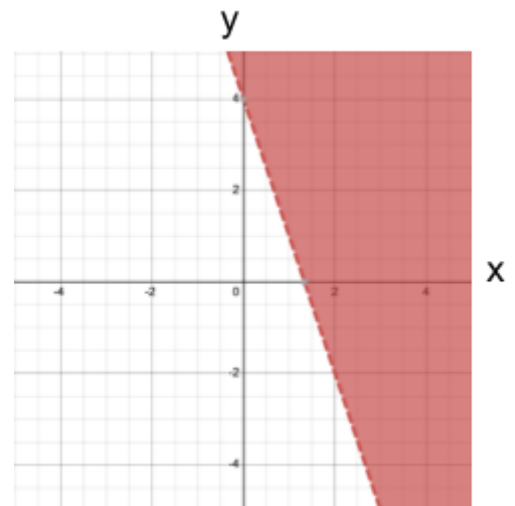
C Level

4. $\frac{12x}{5} - 3x \leq 5.2$

5. $19 - \frac{|x|}{2} < 5$

6. Graph the line $y < 3|x - 1| + 2$

7. From the graph (right) write the equation of the line



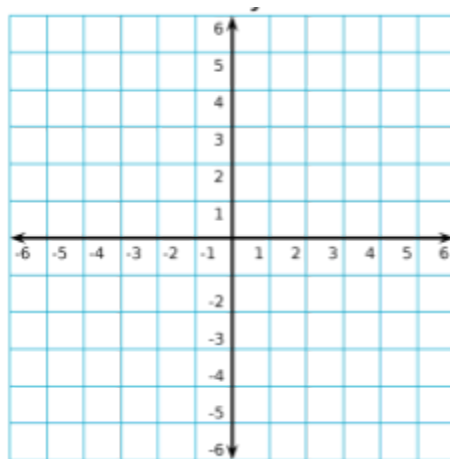
B Level

8. Write an absolute value function that has a vertex at the point (5, 2) and a slope of -1.

9. Solve for x , $\frac{|3.6 - 4x|}{7} \geq 2.5$

10. The school's principal said that if at least $\frac{5}{6}$ of the students vote to have a salad bar in the cafeteria, he will allow it. If y represents the number of students in the school, and 280 votes are enough, how many students attend the school? **Write an inequality** and use it to solve the problem.

11. Use a table of values to graph the equations mentioned in the following problem. Then, answer the question. What is the number of square units in the area of the region bounded by the graphs of $y = 2|x - 2|$ and $y = -|2x - 4| + 4$?



Quadratic Equations (CP and Honors)

D Level

12) For the equation $f(x) = 3(x - 1)^2 - 3$,

a) what is the value of the y-intercept?

b) Will the parabola concave up or down

13) Find all of the solutions for each of the equations

i) $(x - 5)(x + 3) = 0$

ii) $(x - 9)^2 = 0$

iii) $(5x + 2)(3x - 4) = 0$

14) Factor the following quadratic expression

iv) $x^2 + 12x + 36$

v) $x^2 - 81$

C Level

15) Factor and solve the following equations

a. $x^2 - 361 = 0$

b. $6x^2 + 17x + 7 = 0$

16) Solve the following by completing the square. If required keep the answer in simplified radical form.

$$x^2 - 14x + 41 = 0$$

17) Use the quadratic formula to solve the following correct to one decimal place.

18) For the equation $y = x^2 - 6x + 1$ find the coordinates of the turning point.

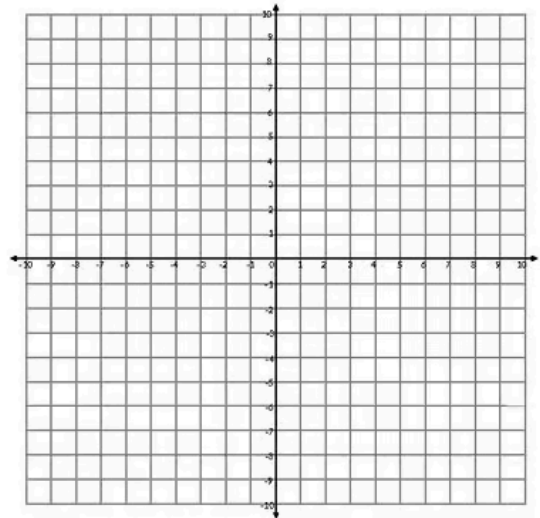
19) What do you know about the discriminant if:

- c. The function is a perfect square?
- d. The graph of the function has two x intercepts?
- e. The graph of the function has no x intercepts?

20) Sketch the following parabolas after calculating the key features including, intercepts and turning points.

f. $f(x) = 3(x - 1)^2 - 4$

g. $y < -2x^2 + x - 15$



B Level

21) Solve the following function with the quadratic formula. Leave answers exact (as a simplified radical or complex number)

$$3x^2 + 6x + 4 = 0$$

22) Find all possible values for b in the following equation

$$x^2 + bx + 28 = 0$$

23) What non-zero integer must be placed in the the second bracket so that the simplified product of these two binomials is also a binomial? $(8x + 4)(4x + _)$

24) Solve the following equation

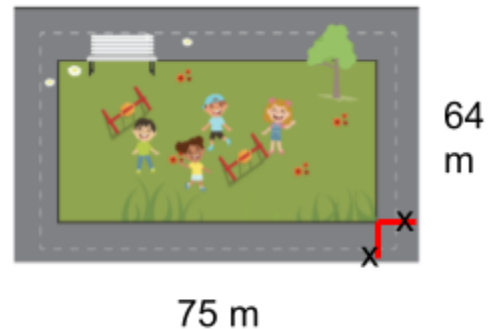
$$\frac{x}{4} + \frac{8}{x} = 3$$

25) The volume of a cylinder is $V = (10x^2 + 60x + 90)\pi$ units cubed. If the height of the cylinder is 10 cm,

a. find an expression for the radius of the cylinder.

26) A town plans to build a new park with a path around the perimeter. The park's land will be 75 m long and 64 m wide, and the path's width will be x m.

- b. Write an equation to find the area of the park inside the path.
- c. Use the equation to find the area of the park if the path is
 - i. 2m wide
 - ii. 3.5m wide



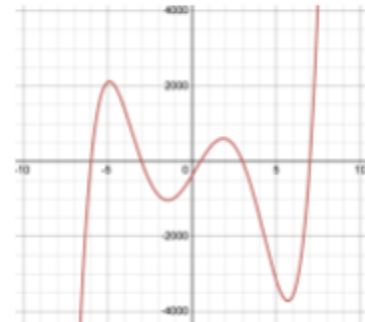
27) If $f(x) = 5.2 - 3.9x$ and $g(x) = 3x^2$, find the value of $f(g(-2.5))$ correct to 2 decimal places

Polynomial Functions (CP and Honors)

D Level

28) Describe the end behaviour of the following function

$$f(x) = 2x^5 - 3x^4 - 101x^3 + 69x^2 + 747x - 378$$



29) Given $f(x) = 2x^3 + 9x^2 + 7x - 6$, calculate the value of:

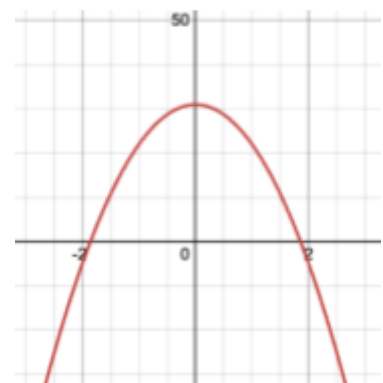
- a. $f(3)$
- b. $f(-2)$
- c. Which value is a factor of $f(x) = 2x^3 + 9x^2 + 7x - 6$

30) Explain how the leading coefficient being negative effects the end behaviour of a function.

C Level

31) Find the domain and range of the following function.

$$f(x) = -9x^2 + 31$$



32) Factor and solve the following equation by finding the highest common factor.

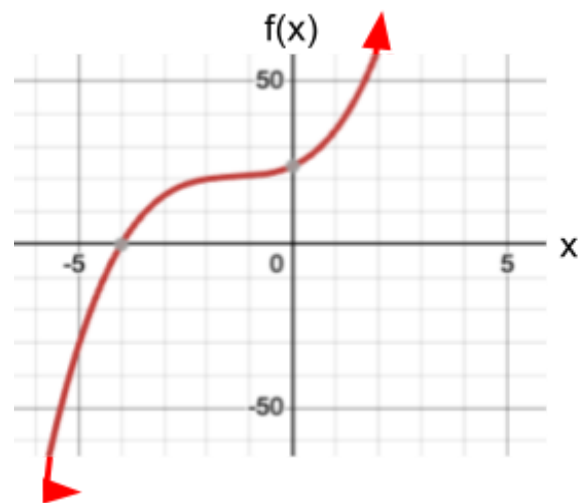
$$2x^4 + 6x^3 - 36x^2 = 0$$

33) Factor and solve the following equation by grouping

$$2x^3 - 3x^2 - 32x + 48 = 0$$

34) Knowing $(x - 3)$ is a factor of $f(x) = 2x^3 - 9x^2 - 11x + 60$, use long division to determine the other quadratic factor

35) A function $f(x) = x^3 + 4x^2 + 6x + 24$ is shown in the graph. Use synthetic division to determine the other (quadratic) factor



36) Using the rational zeros test find all of the possible factors of the function

$$f(x) = 4x^2 + 13x + 3$$

B Level

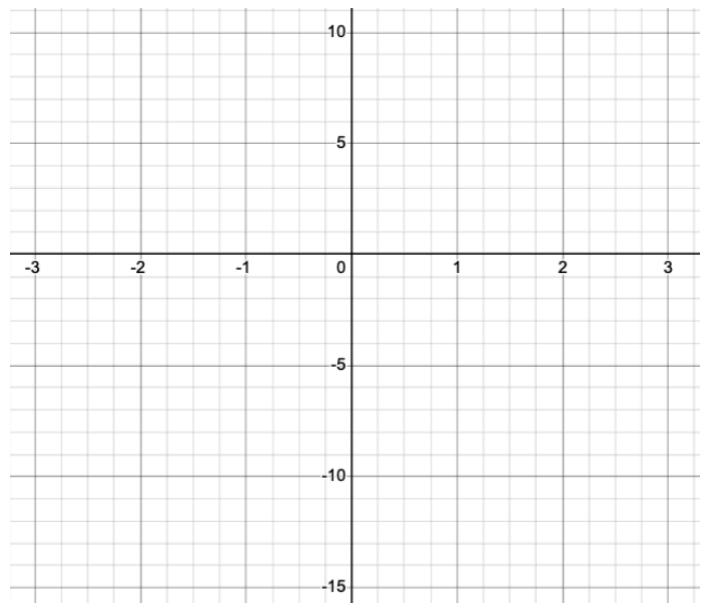
37) Find the real and complex factors for the equation $x^3 - 343 = 0$

38) Write a third degree polynomial that is divisible by $x^2 - 3x - 18$ and has a solution at $x = -2$

39) Suppose a certain species of bird thrives on a small island. The population can be estimated using the function $P(t) = -0.3t^3 + 97t + 800$. Where t is equal to the number of years after 2009.

- What was the population of the birds on the islands in 2009?
- What is the expected population of birds in the year 2024?
- Using the end behaviour of the function, explain why this polynomial model may not be a great predictor of the population over 20 years.
-

40) Find all of the real zeros for the function $f(x) = x^4 - 5x^3 + 7x^2 + 3x - 10$ and use them to sketch the function



41) Find the value of k if the division of $\frac{2x^2 - kx + 2}{x - 2}$ gives a remainder of 6

Polynomial Modelling and Binomial Theorem (Honors only)

D Level

Polynomial Functions - C level skills

C Level

42) For what values of x is the inequality $x^2 + 3x - 10 > 0$ true?

43) I did a quadratic regression for data in a table, which gives me the following data. (4 marks)

x	y	Title	Quadratic Eqn
1	-2.5	Reg Eqn	ax^2+bx+c
2	12	a	1
3	19.3	b	-13
4	11	c	47.7
5	3.4	d	-39.5
6	-4	r^2	0.925
7	12		
8	27.5		

- Write the equation of the polynomial regression line.
- Is this a good regression line representing the data? Why?
- Make a prediction using this regression line for the value of y when $x = 5.5$.
- Was your prediction in part c an example of interpolation or extrapolation?

B Level

47) Determine which values of x make the inequality $x^3 + 4x^2 - 9x - 36 < 0$ true.

48) Expand $(2x + y)^5$

49) A toy manufacturer's total cost of production fits the function $C(x) = 16x + 12500$ and its

revenue function fits the function $R(x) = x^2 - 184x$ where x is the number of toys produced.

- a. Write an inequality for calculating when the company make a profit.
- b. What number of toys is the break-even point for manufacturing these toys?
- c. If they manufacture 350 toys, will they make a profit? If yes, how much?

50) A gardener wants to enclose a rectangular flower bed using 28m of fencing. For what range of lengths will the area be at least 45m^2 ?

51) Expand

d. $(1 + \sqrt{2})^5$

e. $(2 + i)^4$

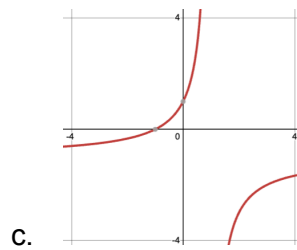
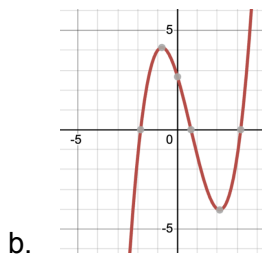
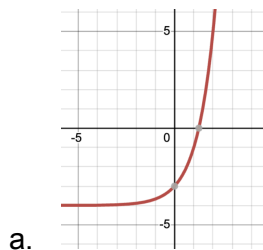
52) The total number of terms in the expansion of $(x + 5)^{12} + (x - 5)^{12}$ after simplification is
(Circle the correct answer)(1 mark)

- f. 13
- g. 6
- h. 7
- i. 26

Rational Functions (CP and Hon)

D Level

53) Which of the following three graphs represents a rational function? Circle the correct answer



54) Simplify the following expression

d.
$$\frac{p+2}{p^2+13p+22}$$

55) Solve for x

e.
$$\frac{3x}{5x+2} = 2$$

f.
$$\frac{4}{3x} + \frac{5}{2x} = 7$$

56) Find the equations for the horizontal and vertical asymptotes for the following functions.

g.
$$f(x) = \frac{1}{x}$$

Vertical Asymptote $x =$

Horizontal Asymptote $y =$

h.
$$g(x) = \frac{1}{x+2} - 1$$

Vertical Asymptote _____

Horizontal Asymptote _____

C Level

57) Simplify the following expression

a.
$$\frac{m^2-4m-45}{m^2+12m+35}$$

58) Find the points of discontinuity for the following functions.

b.
$$g(x) = \frac{1}{x^2-13x+40} - 4$$

Vertical Asymptotes: _____

Horizontal Asymptote: _____

Vertical Asymptotes: _____

Horizontal Asymptote: _____

Holes: _____

c. $h(x) = \frac{x - 3}{x^2 + 6x - 27}$

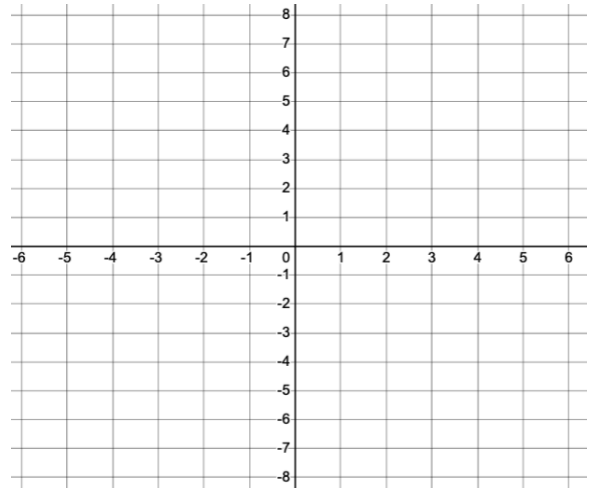
d. $f(x) = \frac{2x + 5}{3x - 4}$

Vertical Asymptotes: _____

Horizontal Asymptote: _____

59) Sketch the following functions showing the asymptotes and y-intercept.

e. $f(x) = \frac{1}{x + 2} - 1$



B Level

60) Solve for x

a. $\frac{x}{3} + \frac{9}{x} = 4$

61) The fire department's hose can fill a swimming pool in 3 hours. The garden hose can fill the pool in 15 hours. How long will it take to fill the pool if both hoses are running?

62) Sketch the following functions showing the asymptotes (with dashed lines)

$f(x) = \frac{1}{x^2 - 2x - 3} + 1$

63) A rare species of insect was discovered in the rainforest of Costa Rica. Environmentalists transplanted the insects into a protected area on March 1 2025. The population of the insect t months after being transplanted is

$$P(t) = \frac{45(1 + 0.6t)}{(3 + 0.02t)}$$

- b. What was the population when $t = 0$?
- c. What will the population be after 10 years?
- d. When will there be 75 insects?

64) The sum of a number and twice its smaller reciprocal $\frac{67}{21}$ is . What is the original number?

Radical Functions (CP and Hon)

D Level

65) Simplify the following radicals (4 marks)

a. $\sqrt{48x}$

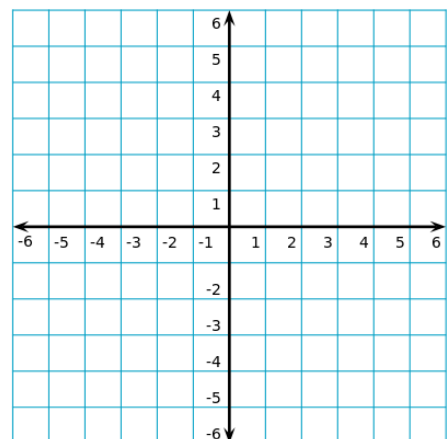
b. $5\sqrt{8x^4y^5}$

c. $\frac{3}{2\sqrt{5}}$

d. $\sqrt[3]{343x^5y^7}$

66) Solve $\sqrt{7x+2} + 5 = 15$

67) Sketch the function $f(x) = \sqrt{x-2} + 1$



68) Rewrite the following exponents in radical form

e. $5^{\frac{1}{2}}$

f. $7^{\frac{4}{3}}$

C Level

69) Solve the following equations

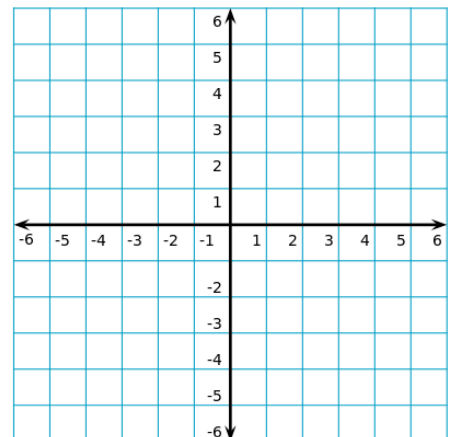
a. $\sqrt[3]{\frac{a^2 - 16}{2}} = 4$

b. $x^{\frac{5}{4}} = 18$

70) Simplify by rationalising the denominator

$$\frac{\sqrt{2}}{3 + \sqrt{5}}$$

71) Sketch the following radical function $f(x) = -3\sqrt{x} + 2$



B Level

72) Simplify the following radical expression

$$\frac{\sqrt[4]{32x^{15}y^9}}{4x^3y^2}$$

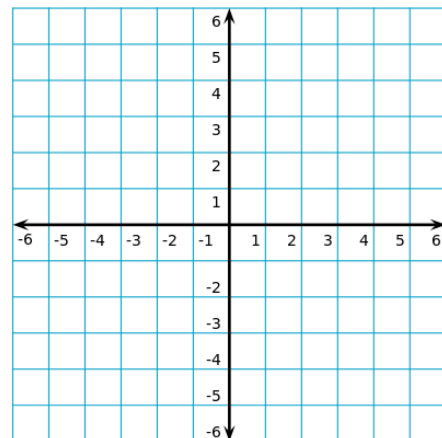
73) Solve $\sqrt{5n + 11} = n + 3$

74) The function $a(h) = 20\sqrt[3]{h - 8.3} + 40$ models the age of a sassafras tree that is h meters high. (4 marks)

- Find the approximate age in years of a sassafras tree that is 2.7m high.
(answer to 1 dec pl)
- Use the model to find the height of the sassafras tree when it was 30 years old
(to the nearest centimeter).

75) Sketch the following function

$$f(x) = -\frac{3}{2}\sqrt{x - 2} + 1$$



Exponential and Logarithmic Functions (Both)

D Level

76) Simplify the expression. (2 marks each)

a. $\frac{x^5 \times (x^3)^0}{x^{-2}}$

b. $(3ab^5 \times 7a^2b^{-2}c)^3$

77) Without graphing, identify if the following represents exponential **growth** or **decay**?

c. $f(x) = 7\left(\frac{1}{4}\right)^x$ _____

d. $g(x) = 8 \times 7^x$ _____

e. $h(x) = \left(\frac{1}{3}\right)^{-x}$ _____

78) Calculate f(5) if

f. $f(x) = 3^x \times x^3$

g. $f(x) = 9.1 \times \left(\frac{1}{2}\right)^x + 2$

h. $f(x) = \log_x 1398$

79) Solve the following equations for x, and show the logarithmic equation you used to solve them.

i. $4^x = 262144$

j. $10^x = 11$

C Level

80) Using the compound interest formula $A = P\left(1 + \frac{r}{k}\right)^{kt}$ find

a. A when P = \$24000, r = 3% pa, t = 10 years and the interest is compounded monthly.

b. P when A = \$38490.58, r = 5.5%, t = 4 and the interest is compounded daily.

81) Solve the following equations

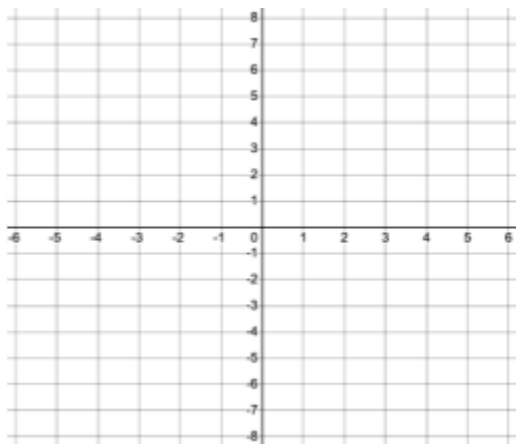
c. $\log_{10}(x^2) = \log_{10}(10 - 3x)$

d. $\log_2 4x = 7$

e. $\log_7(5x + 3) = 3$

82) Complete the table for $f(x) = (2)^x - 3$. Then graph the function

x	$f(x)$
-2	
-1	
0	
1	
2	



83) Identify the domain and range of the function $f(x) = (2)^x - 3$

f. Domain _____

g. Range _____

B Level

84) A medication to treat inflammation in the body has a half-life of 8.6 days. After an infusion a patient has 500mg of medication in their body. How many milligrams of medication will they have in their system when they are due for their next infusion after 8 weeks?

85) The struggling town of Abajo had a population of 50 000 in 2014. It has had a natural population decline of 2.4% per year. What year will the population fall below 20 000 people?

86) For a temperature of 16 degrees Celsius and a height h (in feet) above sea level, the air pressure P (in pounds per square inch - psi) can be modelled by the equation

$P(h) = 17.7e^{-0.0004h}$ and the air density D (in pounds per cubic foot - pcf) can be modelled by the equation $D = 0.0761e^{-0.0004h}$

- What is the air density at 10 000 feet above sea level?
- What is the air pressure at 10 000 feet above sea level?
- What do you notice about the relationship between air pressure and density?
- To the nearest foot, at what height is the air pressure 12 psi?

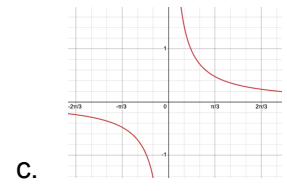
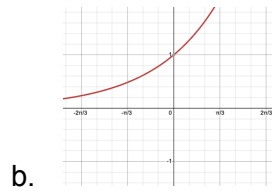
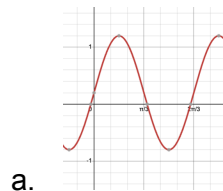
87) A bacteria culture triples every 45 minutes. How long will it take a culture of 10 bacteria to grow to a population of 50 000? Give your answer in hours to the nearest second.

Trigonometry (Honors Only)

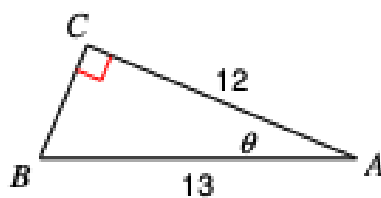
D Level

88)

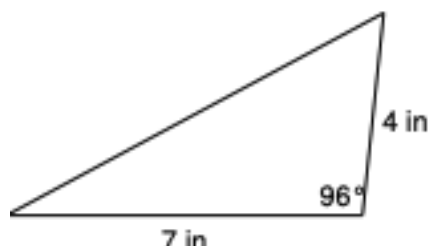
Which of the following three graphs represents a periodic function? Circle the correct answer (1 mark)



89) Find the value of the variable in the following problems (3 marks each)



90)



Calculate the area of the following triangle (3 marks)

91) For the function $f(\theta) = 3 \sin \theta + 14$ find the (5 marks)

a. Amplitude

d. Minimum

b. Midline

e. Period

c. Maximum

92)

Calculate the periods of the following functions (2 marks each)

i. $f(\theta) = \sin 4\theta$

ii. $f(x) = \tan\left(\frac{x}{2}\right) + 5$

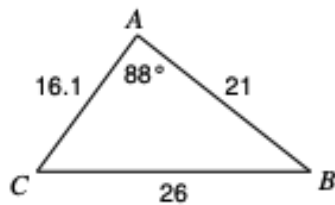
93) Convert from 135° into radians

94) Convert 3 radians to degrees

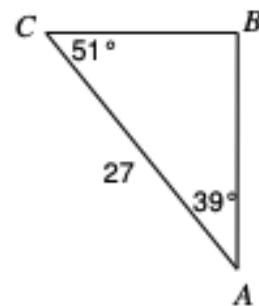
C Level

95) Law of sines (3 marks each)

a. Find the size of $\angle C$

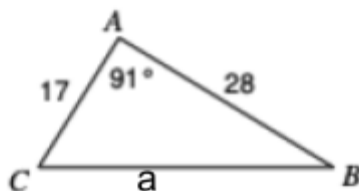


b. Find the length of BC

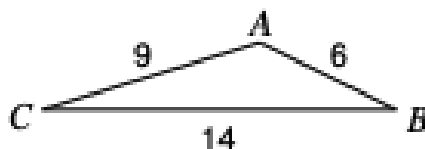


96) Law of cosines (3 marks each)

a. Find the length of a



b. Find the size of $\angle C$



97) Solve trigonometric equations for the given domain (4 marks each)

a. $\sin(3\theta) + 4 = 5 \quad \{0 \leq \theta \leq 2\pi\}$

b. $\sqrt{3} \tan \theta + 1 = 0 \quad \{0 \leq \theta \leq 2\pi\}$

98) Sketch the following trig function showing the period, midline, and maximum and minimum values (4 marks)

a. $f(x) = 3 \sin x + 5 \quad \{0 \leq x \leq 6\pi\}$

B Level

99) Write the equation for a cosine function that has a period of π , an amplitude of 3 and a midline at 2.5

100) Find the size of all three angles of a triangle that has lengths of 4.8cm, 6.1cm and 9.2cm?

101) Solve harder trig equations (6 marks each)

a. $4 \sin^2 \theta = 3 \quad \{0 \leq \theta \leq 2\pi\}$

b. $2 \sin^2 x - 3 \sin x + 1 = 0 \quad \{0 \leq \theta \leq 4\pi\}$

102) Sketch the following trig function showing the period, midline, and maximum and minimum values (4 marks)

$$f(x) = 4 \sin \left(\frac{1}{2}x - \frac{\pi}{2} \right) + 5$$

103) Write a trigonometric function that represents the situation below. (3 marks)

The tide continuously goes in and out each day, raising and lowering a boat (sinusoidally) in the harbour. The boat is only 2 feet above the ocean floor at low tide. And, 6 hours later, at peak high tide, the boat is 26 feet above the ocean floor. Write a cosine function that describes the boat's distance above the ocean floor as it relates to time.

KEY

Precalculus/ Trigonometry Summer Assignment

Name: _____

Welcome to Precalculus/Trigonometry! Going into Precalculus/Trigonometry we will be building off prior concepts that you learned about in previous math courses. Sometimes this can be difficult if you have not reviewed these topics and are proficient in them. These are the topics that you need to review from your previous math course to be successful in Precalculus / Trigonometry . For your summer assignment, you will complete the following pages to review these topics.

Linear Functions (CP Only)

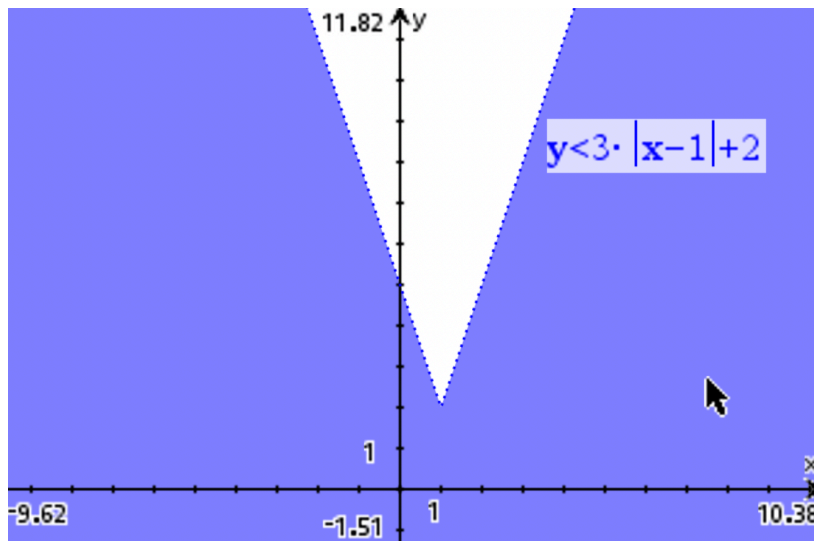
D Level

1. $7x - 22 < 11x + 45$ **Ans: $x > -67/4$**
2. $2|x - 8| = 5$ **Ans: $x = 27/4, 37/4$**
3. If you graphed the function $y = |x - 1| - 4$, what would be the coordinates of the vertex? **Ans: $V(1, -4)$**

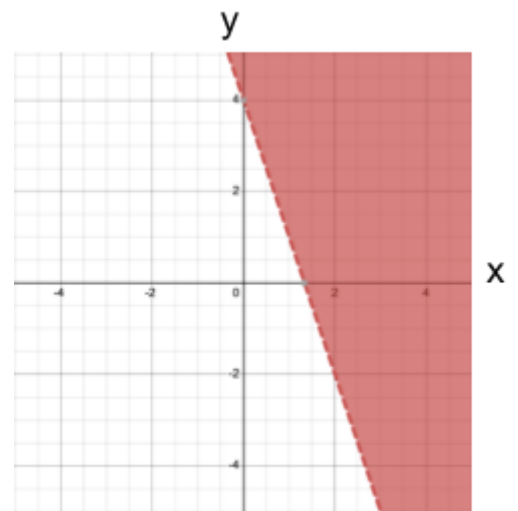
C Level

4. $\frac{12x}{5} - 3x \leq 5.2$ **Ans: $x \leq -26/3$**
5. $19 - \frac{|x|}{2} < 5$ **Ans: $x \leq -28, x \geq 28$**

6. Graph the line $y < 3|x - 1| + 2$



7. From the graph (right) write the equation of the line **Ans: $x > -2x + 4$**



B Level

8. Write an absolute value function that has a vertex at the point (5, 2) and a slope of -1.

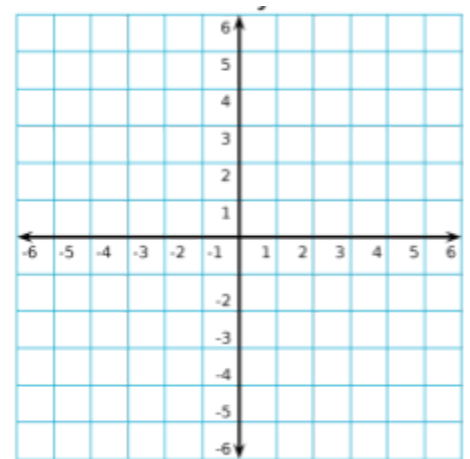
Ans: $y = -|x - 5| + 2$

9. $\frac{|3.6 - 4x|}{7} \geq 2.5$ **Ans: $x \leq -3.475$, $x \geq 5.275$**

10. The school's principal said that if at least $\frac{5}{6}$ of the students vote to have a salad bar in the cafeteria, he will allow it. If y represents the number of students in the school, and 280 votes are enough, how many students attend the school? **Write an inequality** and use it to solve the problem.

Ans: $\frac{5}{6}y \geq 280$, $y \geq 336$

11. Use a table of values to graph the equations mentioned in the following problem. Then, answer the question. What is the number of square units in the area of the region bounded by the graphs of $y = 2|x - 2|$ and $y = -|2x - 4| + 4$?



Ans: Area = 5 u²

Quadratic Equations (CP and Honors)

D Level

12) For the equation $f(x) = 3(x - 1)^2 - 3$,

a) what is the value of the y-intercept? **Ans: (0 , 0)**

b) Will the parabola concave up or down **Ans: concave up**

13) Find all of the solutions for each of the equations

i) $(x - 5)(x + 3) = 0$

Ans: $x = -3, 5$

ii) $(x - 9)^2 = 0$

Ans: $x = 9$

iii) $(5x + 2)(3x - 4) = 0$

Ans: $x = -\frac{2}{5}, \frac{4}{3}$

14) Factor the following quadratic expression

iv) $x^2 + 12x + 36$

v) $x^2 - 81$

Ans: $(x + 6)^2$

Ans: $(x + 9)(x - 9)$

C Level

15) Factor and solve the following equations

a. $x^2 - 361 = 0$

Ans: a. $(x + 19)(x - 19)$, $x = -19, 19$

b. $6x^2 + 17x + 7 = 0$

Ans: $(3x + 7)(2x + 1)$, $x = -7/3, -1/2$

16) Solve the following by completing the square. If required keep the answer in simplified radical form.

Ans: $x = 7 \pm 2\sqrt{2}$

$$x^2 - 14x + 41 = 0$$

17) Use the quadratic formula to solve the following correct to one decimal place.

Ans: $x = -1.2295, 1.8009$

18) For the equation $y = x^2 - 6x + 1$ find the coordinates of the turning point. **Ans: (3, -8)**

19) What do you know about the discriminant if:

- c. The function is a perfect square?
- d. The graph of the function has two x intercepts?
- e. The graph of the function has no x intercepts?

Ans:

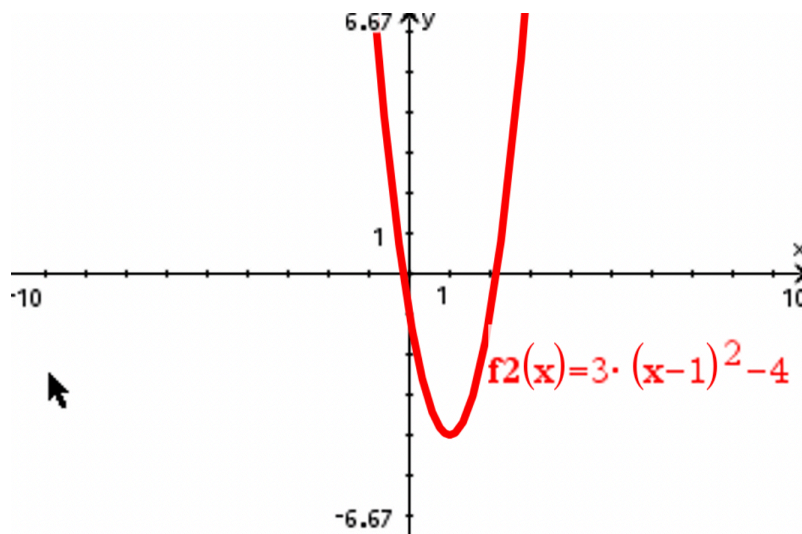
c) Discriminant is a perfect square → 1 rational solution

d) Discriminant > 0 → 2 real solutions

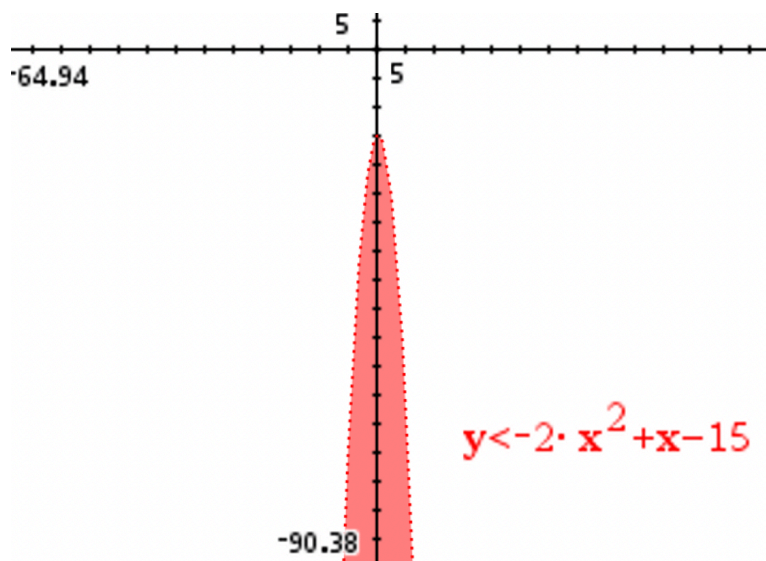
e) Discriminant < 0 → no real solutions

20) Sketch the following parabolas after calculating the key features including, intercepts and turning points.

f. $f(x) = 3(x - 1)^2 - 4$



g. $y < -2x^2 + x - 15$



B Level

21) Solve the following function with the quadratic formula. Leave answers exact (as a simplified radical or complex number)

$$3x^2 + 6x + 4 = 0$$

Ans: $x = (3 \pm \sqrt{13})/2$

22) Find all possible values for b in the following equation

$$x^2 + bx + 28 = 0$$

Ans: Given equation format: use discriminant $b^2 - 4ac = 0$ for one solution, $b = \pm 4\sqrt{7}$

23) What non-zero integer must be placed in the second bracket so that the simplified product of these two binomials is also a binomial? $(8x + 4)(4x + _)$

Ans: $(x + 2)(x + b)$: to make it a binomial, $b = 2$

24) Solve the following equation

$$\frac{x}{4} + \frac{8}{x} = 3$$

Ans: $x = 4, 8$

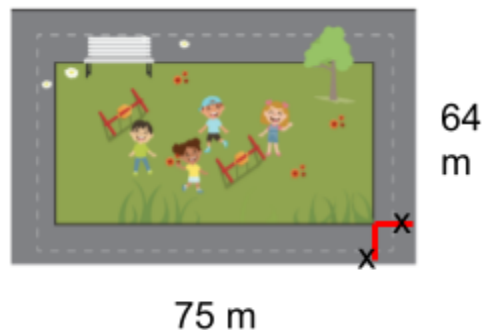
25) The volume of a cylinder is $V = (10x^2 + 60x + 90)\pi$ units cubed. If the height of the cylinder is 10 cm,

a. find an expression for the radius of the cylinder.

$$\text{Ans: } V = \pi r^2 h \rightarrow r = \sqrt{(V / (\pi h))}$$

26) A town plans to build a new park with a path around the perimeter. The park's land will be 75 m long and 64 m wide, and the path's width will be x m.

- b. Write an equation to find the area of the park inside the path.
- c. Use the equation to find the area of the park if the path is
 - i. 2m wide



ii. 3.5m wide

Ans: Area of rectangle inside path = $(75 - 2x)(64 - 2x)$

i) $x = 2 \rightarrow \text{Area} = 71 \times 60 = 4260 \text{ m}^2$

ii) $x = 3.5 \rightarrow \text{Area} = 68 \times 57 = 3876 \text{ m}^2$

27) If $f(x) = 5.2 - 3.9x$ and $g(x) = 3x^2$, find the value of $f(g(-2.5))$ correct to 2 decimal places

Ans: $f(18.75) = -67.93$

Polynomial Functions (CP and Honors)

D Level

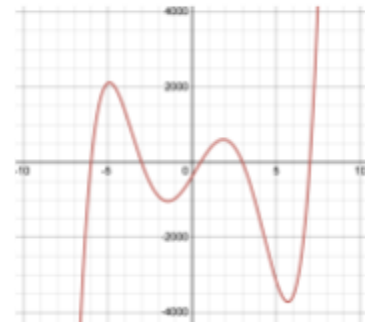
28) Describe the end behaviour of the following function

**Ans: End behavior: Odd degree,
positive leading coefficient $\rightarrow$ as $x \rightarrow \pm\infty$**

as $x \rightarrow -\infty$, $f(x) \rightarrow -\infty$

as $x \rightarrow \infty$, $f(x) \rightarrow \infty$

$$f(x) = 2x^5 - 3x^4 - 101x^3 + 69x^2 + 747x - 378$$



29) Given $f(x) = 2x^3 + 9x^2 + 7x - 6$, calculate the value of:

a. $f(3) = 150$

b. $f(-2) = -72$

c. Which value is a factor of $f(x) = 2x^3 + 9x^2 + 7x - 6$
 $x = -2, -3, \frac{1}{2}$

30) Explain how the leading coefficient being negative effects the end behaviour of a function.

If the leading coefficient is negative the end behavior will be:

as $x \rightarrow -\infty$, $f(x) \rightarrow \infty$, as $x \rightarrow \infty$, $f(x) \rightarrow -\infty$

C Level

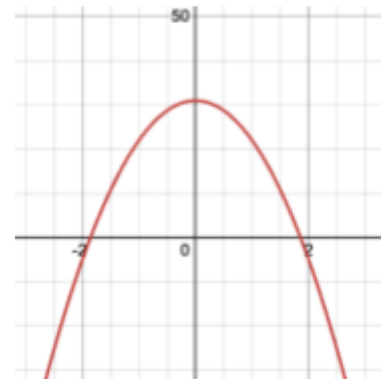
31) Find the domain and range of the following function.

Ans:

Domain $(-\infty, \infty)$

Range $(-\infty, 30]$

$$f(x) = -9x^2 + 31$$



32) Factor and solve the following equation by finding the highest common factor.

$$2x^4 + 6x^3 - 36x^2 = 0$$

Ans: $2x^2(x^2 + 3x - 18) \rightarrow x = -6, 0, 3$

Highest common factor: $2x^2$

33) Factor and solve the following equation by grouping

$$2x^3 - 3x^2 - 32x + 48 = 0$$

Ans: $x = -4, 3/2, 4$

34) Knowing $(x - 3)$ is a factor of $f(x) = 2x^3 - 9x^2 - 11x + 60$, use long division to determine the other quadratic factor

Ans: $x = -5/2, 3, 4$

35) A function $f(x) = x^3 + 4x^2 + 6x + 24$ is shown in the graph. Use synthetic division to determine the other (quadratic) factor



Ans: $x = -4$

36) Using the rational zeros test find all of the possible factors of the function

$$f(x) = 4x^2 + 13x + 3$$

Ans: $x = -3, -\frac{1}{4}$

B Level

37) Find the real and complex factors for the equation $x^3 - 343 = 0$

Ans: Identify: $x^3 - 343 = x^3 - 7^3$

$$(x^3 - 343) = (x - 7)(x^2 + 7x + 49)$$

The real factor is: $x - 7$

Step 3: Find complex roots of the quadratic

$$x = \frac{-7 \pm \sqrt{-147}}{2} = \frac{-7 \pm i\sqrt{147}}{2}$$

Since $\sqrt{147} = \sqrt{49 \cdot 3} = 7\sqrt{3}$:

$$x = \frac{-7 \pm 7i\sqrt{3}}{2}$$

38) Write a third degree polynomial that is divisible by $x^2 - 3x - 18$ and has a solution at $x = -2$

Ans: $f(x) = (x + 1)(x - 2)^2$

39) Suppose a certain species of bird thrives on a small island. The population can be estimated using the function $P(t) = -0.3t^3 + 97t + 800$. Where t is equal to the number of years after 2009.

- What was the population of the birds on the islands in 2009?
- What is the expected population of birds in the year 2024?
- Using the end behaviour of the function, explain why this polynomial model may not be a great predictor of the population over 20 years.

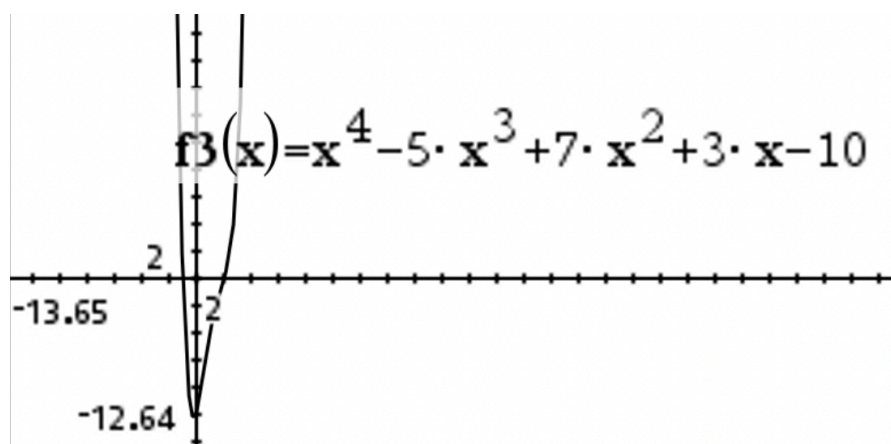
Ans:

a. $P(0) = 800$

b. $P(15) = 1,242.5$

c. Long-term predictions may not be reliable as polynomials don't model limiting behavior

40) Find all of the real zeros for the function $f(x) = x^4 - 5x^3 + 7x^2 + 3x - 10$ and use them to sketch the function **Ans: Real zeros $x = -1, 2$**



41) Find the value of k if the division of $\frac{2x^2 - kx + 2}{x - 2}$ gives a remainder of 6

Ans: $k = 2$

Polynomial Modelling and Binomial Theorem (Honors only)

D Level

Polynomial Functions - C level skills

C Level

42) For what values of x is the inequality $x^2 + 3x - 10 > 0$ true? **Ans: $(-\infty, 5) \cup (2, \infty)$**

43) I did a quadratic regression for data in a table, which gives me the following data. (4 marks)

x	y	Title	Quadratic Eqn
1	-2.5	Reg Eqn	ax^2+bx+c
2	12	a	1
3	19.3	b	-13
4	11	c	47.7
5	3.4	d	-39.5
6	-4	r^2	0.925
7	12		
8	27.5		

- a. Write the equation of the polynomial regression line.

Ans: $y = x^2 - 13x + 47.7$

- b. Is this a good regression line representing the data? Why?

Ans: r^2 , the coefficient of determination, tells us how much of the variation in the dependent variable is explained by the model.

An r^2 of 0.92 means 92% of the variation in the data is explained by the regression line, indicating a very good fit of the regression line to the data.

- c. Make a prediction using this regression line for the value of y when $x = 5.5$.

Ans: $f(5.5) = 6.45$

- d. Was your prediction in part c an example of interpolation or extrapolation?

Ans: Interpolation

Interpolation is when you use a model (like a regression line) to **predict values within the range of your existing data**.

It's generally **reliable** because it's based on patterns you've already observed.

B Level

47) Determine which values of x make the inequality $x^3 + 4x^2 - 9x - 36 < 0$ true.

Ans: $(x+4)(x+3)(x-3) < 0$, $x = (-\infty, -4) \cup (-3, 3)$

48) Expand $(2x + y)^5$ **Ans: $8x^5 + 20x^4y + 15x^3y^2 + 5x^2y^3 + y^5$**

49) A toy manufacturer's total cost of production fits the function $C(x) = 16x + 12500$ and its revenue function fits the function $R(x) = x^2 - 184x$ where x is the number of toys produced.

a. Write an inequality for calculating when the company make a profit.

A company makes a **profit** when:

$$\text{Revenue} > \text{Cost} \Rightarrow R(x) > C(x)$$

Substitute the given functions:

$$x^2 - 184x > 16x + 12,500$$

Move all terms to one side:

$$x^2 - 184x - 16x - 12,500 > 0 \Rightarrow x^2 - 200x - 12,500 > 0$$

b. What number of toys is the break-even point for manufacturing these toys?

The break-even point occurs when $R(x) = C(x)$. Set the functions equal:

$$x^2 - 184x = 16x + 12,500$$

Bring all terms to one side:

$$x^2 - 200x - 12,500 = 0$$

Solve using the quadratic formula:

$$x = \frac{-(-200) \pm \sqrt{(-200)^2 - 4(1)(-12,500)}}{2(1)} = \frac{200 \pm \sqrt{40,000 + 50,000}}{2}$$

$$x = \frac{200 \pm \sqrt{90,000}}{2} \Rightarrow \sqrt{90,000} = 300$$

$$x = \frac{200 \pm 300}{2} \Rightarrow x = \frac{500}{2} = 250, \quad x = \frac{-100}{2} = -50$$

We discard $x = -50$ (not meaningful in this context).

► **Break-even point:** $x = 250$ toys

- c. If they manufacture 350 toys, will they make a profit? If yes, how much?

Check:

- $R(350) = 350^2 - 184(350) = 122,500 - 64,400 = 58,100$
- $C(350) = 16(350) + 12,500 = 5,600 + 12,500 = 18,100$

Now compare:

$$\text{Profit} = R(350) - C(350) = 58,100 - 18,100 = \boxed{40,000}$$

50) A gardener wants to enclose a rectangular flower bed using 28m of fencing. For what range of lengths will the area be at least 45m^2 ?

51) Expand

d. $(1 + \sqrt{2})^5$ **Ans: $41 + \text{sqrt}(2)$**

e. $(2 + i)^4$ **Ans: -7 + 24i**

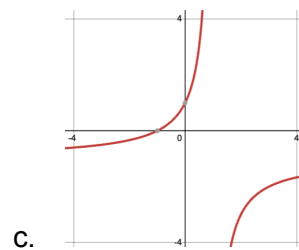
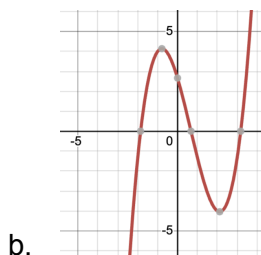
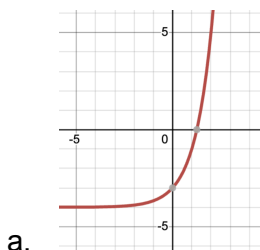
52) The total number of terms in the expansion of $(x + 5)^{12} + (x - 5)^{12}$ after simplification is
(Circle the correct answer)(1 mark)

- f. 13
- g. 6
- h. 7 → Ans**
- i. 26

Rational Functions (CP and Hon)

D Level

53) Which of the following three graphs represents a rational function? Circle the correct answer **Ans: c**



54) Simplify the following expression

d. $\frac{p + 2}{p^2 + 13p + 22}$

Ans: $1/(p+11)$

55) Solve for x

e. $\frac{3x}{5x + 2} = 2$

Ans: $x = -4/7$

f. $\frac{4}{3x} + \frac{5}{2x} = 7$

Ans: $x = 23/42$

56) Find the equations for the horizontal and vertical asymptotes for the following functions.

g. $f(x) = \frac{1}{x}$

Vertical Asymptote **$x = 0$**

Horizontal Asymptote **$y = 0$**

h. $g(x) = \frac{1}{x+2} - 1$

Vertical Asymptote **$x = -2$**

Horizontal Asymptote **$y = -1$**

C Level

57) Simplify the following expression

a. $\frac{m^2 - 4m - 45}{m^2 + 12m + 35}$

Ans: $(m - 9) / (m + 7)$

58) Find the points of discontinuity for the following functions.

b. $g(x) = \frac{1}{x^2 - 13x + 40} - 4$

Ans: VA: $x = 5, 8$, HA: $y = -4$

c. $h(x) = \frac{x - 3}{x^2 + 6x - 27}$

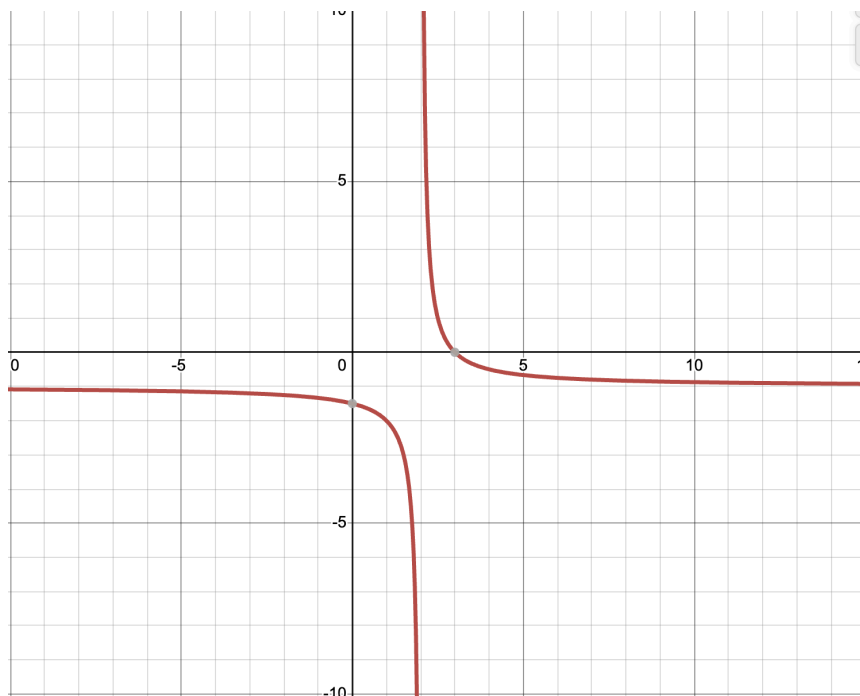
Ans: VA: $x = -9, 8$, HA: $y = 0$, Hole $x = 3$

d. $f(x) = \frac{2x + 5}{3x - 4}$

Ans: VA: $x = 4/3$, HA: $y = 2/3$

59) Sketch the following functions showing the asymptotes and y-intercept.

e. $f(x) = \frac{1}{x+2} - 1$



B Level

60) Solve for x

a. $\frac{x}{3} + \frac{9}{x} = 4$

Ans: x = 3, 9

61) The fire department's hose can fill a swimming pool in 3 hours. The garden hose can fill the pool in 15 hours. How long will it take to fill the pool if both hoses are running?

Final Answer:

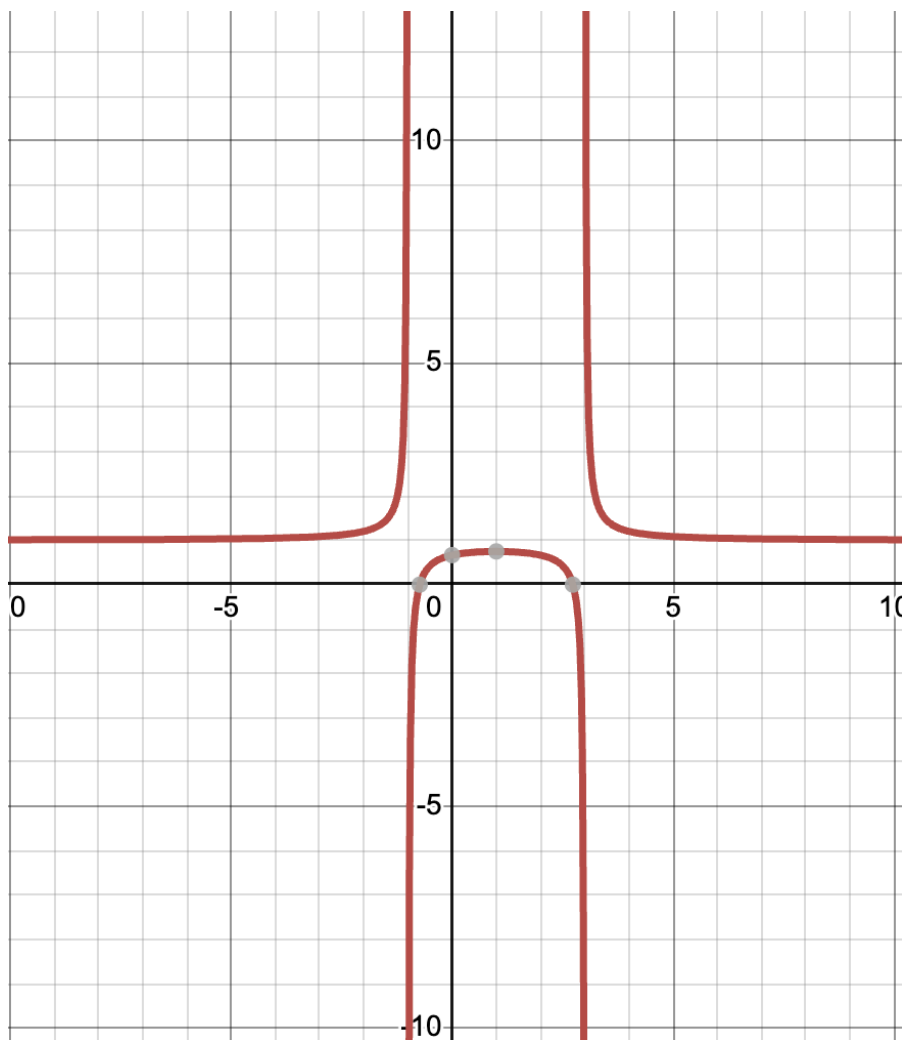
2.5 hours

or

2 hours and 30 minutes

62) Sketch the following functions showing the asymptotes (with dashed lines)

$$f(x) = \frac{1}{x^2 - 2x - 3} + 1$$



63) A rare species of insect was discovered in the rainforest of Costa Rica. Environmentalists transplanted the insects into a protected area on March 1 2025. The population of the insect t months after being transplanted is

$$P(t) = \frac{45(1 + 0.6t)}{(3 + 0.02t)}$$

b. What was the population when $t = 0$?

formula:

$$P(0) = \frac{45(1 + 0.6 \cdot 0)}{3 + 0.02 \cdot 0} = \frac{45 \cdot 1}{3} = \frac{45}{3} = \boxed{15}$$

- c. What will the population be after 10 years?

Since t is in months, 10 years = $10 \times 12 = 120$ months

$$P(120) = \frac{45(1 + 0.6 \cdot 120)}{3 + 0.02 \cdot 120} = \frac{45(1 + 72)}{3 + 2.4} = \frac{45 \cdot 73}{5.4}$$

$$= \frac{3285}{5.4} \approx \boxed{608.33}$$

So, after 10 years, the population is approximately **608 insects**.

- d. When will there be 75 insects?

Set $P(t) = 75$:

$$\frac{45(1 + 0.6t)}{3 + 0.02t} = 75$$

Multiply both sides by $3 + 0.02t$:

$$45(1 + 0.6t) = 75(3 + 0.02t)$$

Expand both sides:

$$45 + 27t = 225 + 1.5t$$

Now solve:

$$27t - 1.5t = 225 - 45 \Rightarrow 25.5t = 180 \Rightarrow t = \frac{180}{25.5} \approx \boxed{7.06 \text{ months}}$$

- 64) The sum of a number and twice its smaller reciprocal $\frac{67}{21}$ is . What is the original number?
Ans: $x + 2(1/x) = 67/21$, $x = 7/3$ or $6/7$

Radical Functions (CP and Hon)

D Level

- 65) Simplify the following radicals (4 marks)

a. $\sqrt{48x}$ **Ans: $4\sqrt{3x}$**

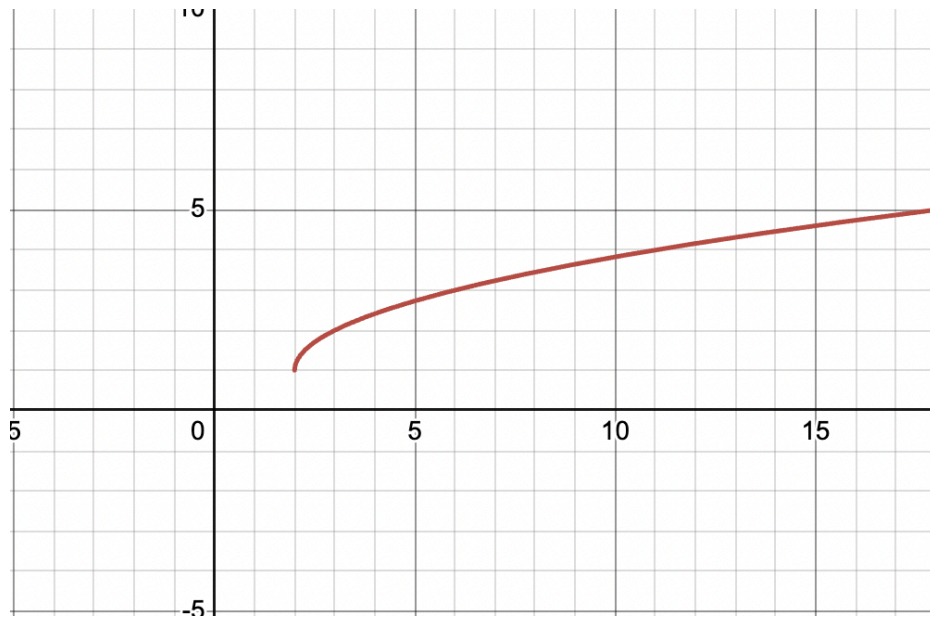
b. $5\sqrt{8x^4y^5}$ **Ans: $10x^2y^2\sqrt{2y}$**

c. $\frac{3}{2\sqrt{5}}$ **Ans: $3\sqrt{5}/10$**

d. $\sqrt[3]{343x^5y^7}$ **Ans: $47xy^2\sqrt[3]{x^2y}$ $\sqrt[3]{} = \text{cubic root}$**

66) Solve $\sqrt{7x+2} + 5 = 15$ **Ans: $x = 14$**

67) Sketch the function $f(x) = \sqrt{x-2} + 1$



68) Rewrite the following exponents in radical form

e. $5^{\frac{1}{2}}$

$\sqrt{5}$

f. $7^{\frac{4}{3}}$

$\sqrt[3]{7^4}$

C Level

69) Solve the following equations

a. $\sqrt[3]{\frac{a^2 - 16}{2}} = 4$

Ans: a = - 12, 12

b. $x^{\frac{5}{4}} = 18$

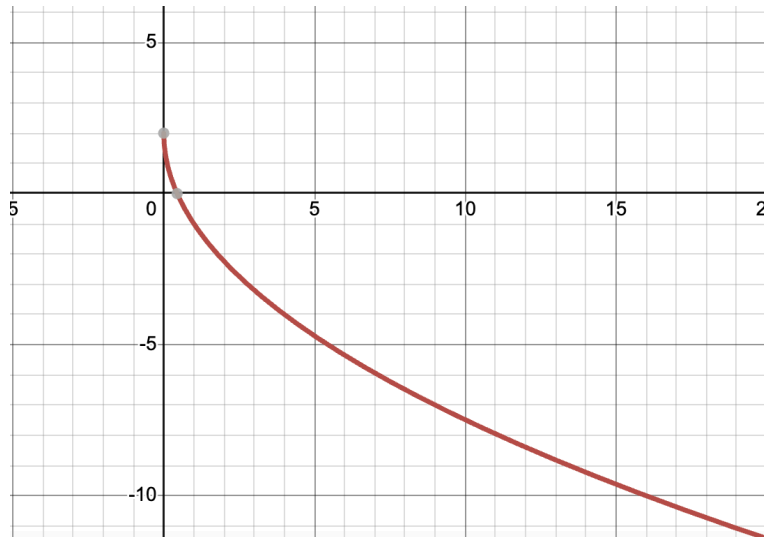
Ans: x = $18^{\frac{4}{5}}$ or 10.097

70) Simplify by rationalising the denominator

$$\frac{\sqrt{2}}{3 + \sqrt{5}}$$

Ans:
$$\frac{(3\sqrt{2} - \sqrt{10})}{4}$$

71) Sketch the following radical function $f(x) = -3\sqrt{x} + 2$



B Level

72) Simplify the following radical expression

$$\frac{\sqrt[4]{32x^{15}y^9}}{4x^3y^2}$$

$$\frac{\sqrt[4]{2 \cdot x^3 \cdot y^1}}{2}$$

Ans:

73) Solve $\sqrt{5n + 11} = n + 3$

Ans: n = - 2, -1

74) The function $a(h) = 20\sqrt[3]{h - 8.3} + 40$ models the age of a sassafras tree that is h meters high. (4 marks)

- Find the approximate age in years of a sassafras tree that is 2.7m high.
(answer to 1 dec pl)

Substitute $h = 2.7$:

$$a(2.7) = 20(2.7 - 8.3)^{1/3} + 40 = 20(-5.6)^{1/3} + 40$$

First compute the cube root of -5.6:

$$(-5.6)^{1/3} \approx -1.77$$

Now plug it in:

$$a(2.7) \approx 20(-1.77) + 40 = -35.4 + 40 = \boxed{4.6 \text{ years}}$$

- b. Use the model to find the height of the sassafras tree when it was 30 years old (to the nearest centimeter).

We're given:

$$a(h) = 30 = 20(h - 8.3)^{1/3} + 40$$

Subtract 40:

$$-10 = 20(h - 8.3)^{1/3}$$

Divide by 20:

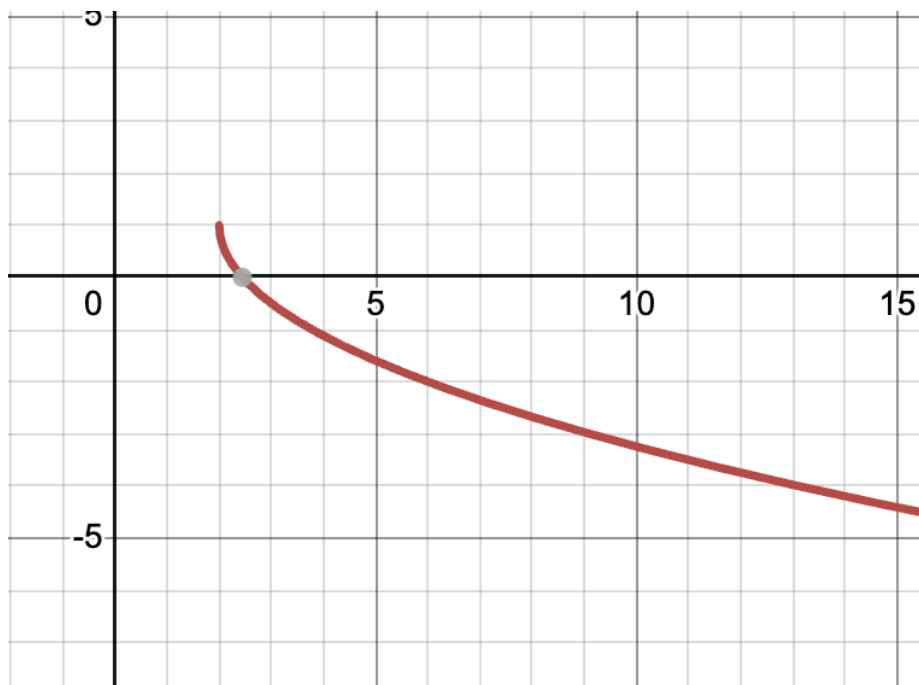
$$(h - 8.3)^{1/3} = -0.5$$

Cube both sides:

$$h - 8.3 = (-0.5)^3 = -0.125 \Rightarrow h = 8.3 - 0.125 = \boxed{8.18 \text{ meters}} = \boxed{818 \text{ cm}}$$

75) Sketch the following function

$$f(x) = -\frac{3}{2}\sqrt{x-2} + 1$$



Exponential and Logarithmic Functions (Both)

D Level

76) Simplify the expression. (2 marks each)

a. $\frac{x^5 \times (x^3)^0}{x^{-2}}$

Ans: x^7

b. $(3ab^5 \times 7a^2b^{-2}c)^3$

Ans: $9261 a^9 b^9 c^3$

77) Without graphing, identify if the following represents exponential **growth** or **decay**?

c. $f(x) = 7\left(\frac{1}{4}\right)^x$ **decay**

d. $g(x) = 8 \times 7^x$ **growth**

e. $h(x) = \left(\frac{1}{3}\right)^{-x}$ **none**

78) Calculate $f(5)$ if

f. $f(x) = 3^x \times x^3$ **Ans: 30,375**

- g. $f(x) = 9.1 \times \left(\frac{1}{2}\right)^x + 2$ **Ans: 2.28**
 h. $f(x) = \log_x 1398$ **Ans: 4.5**

79) Solve the following equations for x, and show the logarithmic equation you used to solve them.

i. $4^x = 262144$

Ans: x = 9

j. $10^x = 11$

Ans: 1.04

C Level

80) Using the compound interest formula $A = P\left(1 + \frac{r}{k}\right)^{kt}$ find

- a. A when $P = \$24000$, $r = 3\%$ pa, $t = 10$ years and the interest is compounded monthly. **Ans: \$32,384.50**

- b. P when $A = \$38490.58$, $r = 5.5\%$, $t = 4$ and the interest is compounded daily. **Ans: \$31,513.80**

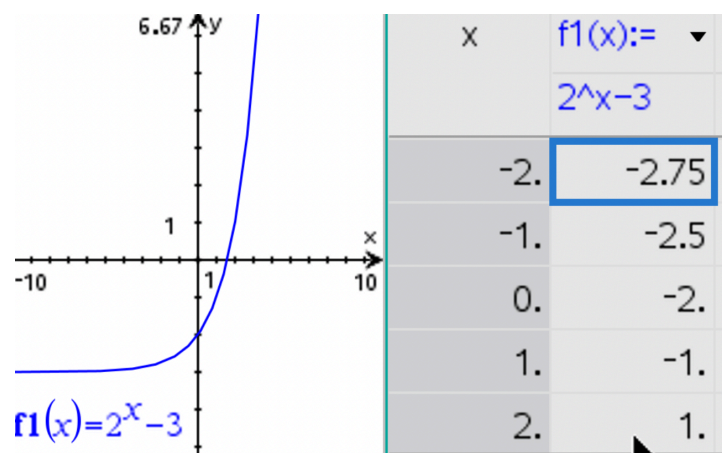
81) Solve the following equations

c. $\log_{10}(x^2) = \log_{10}(10 - 3x)$ **Ans: x = -5, 2**

d. $\log_2 4x = 7$ **Ans: x = 32**

e. $\log_7(5x + 3) = 3$ **Ans: x = 68**

82) Complete the table for $f(x) = (2)^x - 3$. Then graph the function



- 83) Identify the domain and range of the function $f(x) = (2)^x - 3$
- f. Domain **all real numbers** g. Range **$(-3, \infty)$**

B Level

84) A medication to treat inflammation in the body has a half-life of 8.6 days. After an infusion a patient has 500mg of medication in their body. How many milligrams of medication will they have in their system when they are due for their next infusion after 8 weeks?

$$A(t) = A_0 \left(\frac{1}{2} \right)^{t/h}$$

Where:

- $A_0 = 500$ mg (initial amount)
- $h = 8.6$ days (half-life)
- $t = 8 \times 7 = 56$ days (time until next infusion)
- $A(t)$ = amount of medication remaining

Step 2: Plug in values

$$A(56) = 500 \left(\frac{1}{2} \right)^{\frac{56}{8.6}} = 500 \cdot \left(\frac{1}{2} \right)^{6.5116}$$
$$A(56) \approx 500 \cdot 0.0106 \approx \boxed{5.3 \text{ mg}}$$

Final Answer:

After 8 weeks, about 5.3 milligrams of the medication remains in the patient's system.

85) The struggling town of Abajo had a population of 50 000 in 2014. It has had a natural population decline of 2.4% per year. What year will the population fall below 20 000 people?

$$P(t) = P_0(1 - r)^t$$

$$P(t) = 50,000(0.976)^t$$

$$50,000(0.976)^t < 20,000$$

$$(0.976)^t < \frac{20,000}{50,000} = 0.4$$

Take logarithms of both sides:

$$t \cdot \log(0.976) < \log(0.4)$$

$$t > \frac{\log(0.4)}{\log(0.976)} \approx \frac{-0.3979}{-0.0105} \approx 37.89$$

$$2014 + 38 = \boxed{2052}$$

The population will fall below 20,000 in the year $\boxed{2052}$.

86) For a temperature of 16 degrees Celsius and a height h (in feet) above sea level, the air pressure P (in pounds per square inch - psi) can be modelled by the equation

$P(h) = 17.7e^{-0.0004h}$ and the air density D (in pounds per cubic foot - pcf) can be modelled by the equation $D = 0.0761e^{-0.0004h}$

a. What is the air density at 10 000 feet above sea level?

Use:

$$D(h) = 0.0761e^{-0.0004h}$$

Substitute $h = 10,000$:

$$D(10,000) = 0.0761 \cdot e^{-0.0004 \cdot 10000} = 0.0761 \cdot e^{-4} \approx 0.0761 \cdot 0.0183 \approx \boxed{0.00139 \text{ pcf}}$$

b. What is the air pressure at 10 000 feet above sea level?

Use:

$$P(h) = 17.7e^{-0.0004h}$$

$$P(10,000) = 17.7 \cdot e^{-4} \approx 17.7 \cdot 0.0183 \approx \boxed{0.324 \text{ psi}}$$

c. What do you notice about the relationship between air pressure and d

✓ Both **air pressure** and **air density**:

- Decrease **exponentially** with height
- Use the same decay rate (-0.0004)
- Follow **similar curves**, maintaining a proportional relationship

🧠 **Conclusion:** Air pressure and air density are **directly proportional** in this model — they decrease together at the same exponential rate.

Density?

d. To the nearest foot, at what height is the air pressure 12 psi?

Use:

$$12 = 17.7e^{-0.0004h}$$

Solve for h :

Divide both sides:

$$\begin{aligned} \frac{12}{17.7} &= e^{-0.0004h} \Rightarrow \ln\left(\frac{12}{17.7}\right) = -0.0004h \\ \ln(0.67797) &\approx -0.389 \Rightarrow h = \frac{-0.389}{-0.0004} = 972.5 \end{aligned}$$

87) A bacteria culture triples every 45 minutes. How long will it take a culture of 10 bacteria to grow to a population of 50 000? Give your answer in hours to the nearest second.

$$P(t) = P_0 \cdot a^{t/T}$$

Where:

- $a = 3$ (tripling factor)
- $T = 45$ minutes (tripling every 45 minutes)
- t = time in **minutes**

$$50,000 = 10 \cdot 3^{t/45}$$

Step 2: Solve for t

Divide both sides:

$$5,000 = 3^{t/45}$$

Take logarithms of both sides:

$$\log(5,000) = \frac{t}{45} \cdot \log(3) \Rightarrow \frac{t}{45} = \frac{\log(5,000)}{\log(3)} \approx \frac{3.69897}{0.4771} \approx 7.75$$

$$t \approx 7.75 \cdot 45 \approx 348.75 \text{ minutes}$$

Step 3: Convert minutes to hours, minutes, seconds

$$348.75 \text{ minutes} = 5 \text{ hours}, 48 \text{ minutes}, 45 \text{ seconds}$$

Final Answer:

It will take approximately 5 hours, 48 minutes, 45 seconds for the bacteria culture to grow from 10 to 50,000.