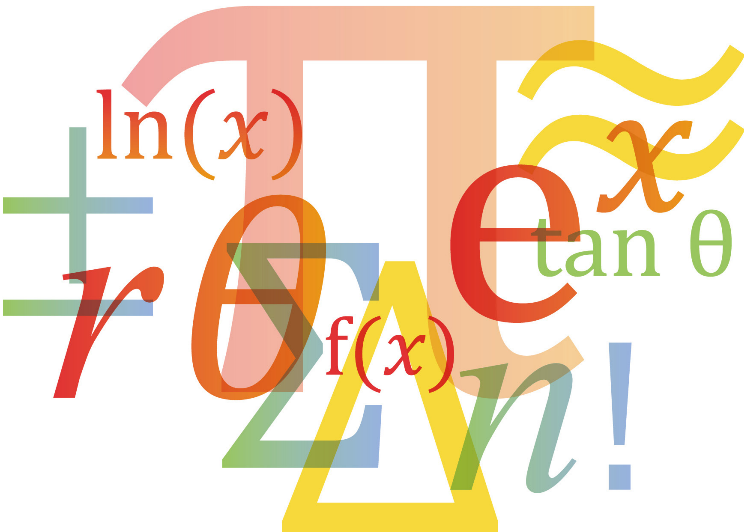
Cambridge Assessment International Education

Specimen Paper Answers – Paper 1

Cambridge International AS & A Level Mathematics 9709

For examination from 2020



Version 2

Cambridge Pathway

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Contents

Introduction

The main aim of this booklet is to exemplify standards for those teaching Cambridge International AS & A Level Mathematics 9709, and to show examples of model answers to the 2020 Specimen Paper 1. Paper 1 assesses the syllabus content for Pure Mathematics 1. We have provided answers for each question in the specimen paper, along with examiner comments explaining where and why marks were awarded. Candidates need to demonstrate the appropriate techniques, as well as applying their knowledge when solving problems.

Individual examination questions may involve ideas and methods from more than one section of the syllabus content for that component. The main focus of examination questions will be the AS & A Level Mathematics subject content. However, candidates may need to make use of prior knowledge and mathematical techniques from previous study, as listed in the introduction to section 3 of the syllabus.

There are 10 to 12 structured questions in Paper 1; candidates must answer **all** questions. Questions are of varied lengths and often contain several parts, labelled (a), (b), (c), which may have sub-parts (i), (ii), (iii), as needed. Some questions might require candidates to sketch graphs or diagrams, or draw accurate graphs.

Candidates are expected to answer directly on the question paper. All working should be shown neatly and clearly in the spaces provided for each question. New questions often start on a fresh page, so more answer space may be provided than is needed. If additional space is required, candidates should use the lined page at the end of the question paper, where the question number or numbers must be clearly shown.

The mark schemes for the Specimen Papers are available to download from the School Support Hub at www.cambridgeinternational.org/support

2020 Specimen Mark Scheme 1

Past exam resources and other teacher support materials are available on the School Support Hub (www.cambridgeinternational.org/support).

Assessment overview

There are three routes for Cambridge International AS & A Level Mathematics. Candidates may combine components as shown below.

Route 1 **AS Level only** (Candidates take the AS components in the same series)	**Paper 1** **Pure Mathematics 1**	**Paper 2** **Pure Mathematics 2**	**Paper 3** **Pure Mathematics 3**	**Paper 4** **Mechanics**	**Paper 5** **Probability & Statistics 1**	**Paper 6** **Probability & Statistics 2**
Either	✓		Not available for AS Level		✓	Not available for AS Level
Or	✓			✓		
Or Note this option in Route 1 cannot count towards A Level	✓	✓				

Route 2 **A Level (staged over two years)**	**Paper 1** **Pure Mathematics 1**	**Paper 2** **Pure Mathematics 2**	**Paper 3** **Pure Mathematics 3**	**Paper 4** **Mechanics**	**Paper 5** **Probability & Statistics 1**	**Paper 6** **Probability & Statistics 2**
Either Year 1 AS Level	✓	Not available for A Level		✓		
Year 2 Complete the A Level			✓		✓	
Or Year 1 AS Level	✓				✓	
Year 2 Complete the A Level			✓			✓
Or Year 1 AS Level	✓				✓	
Year 2 Complete the A Level			✓	✓		

Route 3 **A Level** (Candidates take the A Level components in the same series)	**Paper 1** **Pure Mathematics 1**	**Paper 2** **Pure Mathematics 2**	**Paper 3** **Pure Mathematics 3**	**Paper 4** **Mechanics**	**Paper 5** **Probability & Statistics 1**	**Paper 6** **Probability & Statistics 2**
Either	✓	Not available for A Level	✓	✓	✓	
Or	✓		✓		✓	✓

Paper 1 – Pure Mathematics 1

- Written examination, 1 hour 50 minutes, 75 marks
- 10 to 12 structured questions based on the Pure Mathematics 1 subject content
- Candidates answer all questions
- Externally assessed by Cambridge International
- 60% of the AS Level
- 30% of the A Level

This is compulsory for AS Level and A Level.

Assessment objectives

The assessment objectives (AOs) are the same for all papers:

AO1 Knowledge and understanding

- Show understanding of relevant mathematical concepts, terminology and notation
- Recall accurately and use appropriate mathematical manipulative techniques

AO2 Application and communication

- Recognise the appropriate mathematical procedure for a given situation
- Apply appropriate combinations of mathematical skills and techniques in solving problems
- Present relevant mathematical work, and communicate corresponding conclusions, in a clear and logical way

Weightings for assessment objectives

The approximate weightings (± 5%) allocated to each of the AOs are summarised below.

Assessment objectives as an approximate percentage of each component

Assessment objective	Weighting in components %					
	Paper 1	Paper 2	Paper 3	Paper 4	Paper 5	Paper 6
AO1 Knowledge and understanding	55	55	45	55	55	55
AO2 Application and communication	45	45	55	45	45	45

Assessment objectives as an approximate percentage of each qualification

Assessment objective	Weighting in AS Level %	Weighting in A Level %
AO1 Knowledge and understanding	55	52
AO2 Application and communication	45	48

Question 1

The allocation of marks is indicated using red circles, e.g. B1 shows where a B1 mark has been awarded.

1 The following points

$$A(0, 1), B(1, 6), C(1.5, 7.75), D(1.9, 8.79) \text{ and } E(2, 9)$$

lie on the curve $y = f(x)$. The table below shows the gradients of the chords AE and BE.

Chord	AE	BE	CE	DE
Gradient of chord	4	3	2.5 B1	2.1 B1

(a) Complete the table to show the gradients of CE and DE. [2]

For CE gradient $= \dfrac{9-7.75}{2-1.5} = \dfrac{1.25}{0.5} = 2.5$

For DE gradient $= \dfrac{9-8.79}{2-1.9} = \dfrac{0.21}{0.1} = 2.1$

Examiner comment

Working is not required in this case for the marks to be awarded.

(b) State what the values in the table indicate about the value of $f'(2)$. [1]

$f'(2)$ gives the value of the gradient of the curve at $x = 2$

The values in the table indicate that the gradients of the chords tends towards 2. B1

Examiner comment

The mark is awarded for stating $f'(2) = 2$; again, working is not required to be shown for full marks to be awarded in this question.

Question 2

2 Functions f and g are defined by

$$f : x \mapsto 3x + 2,\ x \in \mathbb{R},$$

$$g : x \mapsto 4x - 12,\ x \in \mathbb{R}.$$

Solve the equation $f^{-1}(x) = gf(x)$. [4]

Let $f(x) = y$: $y = 3x + 2$

$y - 2 = 3x$

$x = \frac{y-2}{3}$

$\therefore f^{-1}(x) = \frac{x-2}{3}$ B1

$gf(x) = 4(3x + 2) - 12$ B1

$= 12x - 4$

When $\frac{x-2}{3} = 12x - 4$ M1

$x - 2 = 36x - 12$

$10 = 35x$

$x = \frac{10}{35} = \frac{2}{7}$ A1

Examiner comment

Candidates should show their working here, as there is a method mark awarded for equating *their* $f^{-1}(x)$ and $gf(x)$ expressions.

Question 3

3 An arithmetic progression has first term 7. The nth term is 84 and the $(3n)$th term is 245.

Find the value of n. [4]

$u_n = a + (n-1)d$ and $a = 7$.

$\therefore\ 84 = 7 + (n-1)d$ and $245 = 7 + (3n-1)d$ **B1**

So $77 = (n-1)d$ and $238 = (3n-1)d$ **B1**

$$\therefore \frac{77}{n-1} = \frac{238}{3n-1}$$ **M1**

$$231n - 77 = 238n - 238$$

$$161 = 7n$$

$$n = 23$$ **A1**

Examiner comment

The first B mark is for *either* of the expressions for the *n*th or (3*n*)th terms being correct. The method mark would be awarded for any correct method to eliminate *d*. If *n* is eliminated first, the mark is only awarded when *d* is used to find *n*.

Question 4

4 A curve has equation $y = \mathrm{f}(x)$. It is given that $\mathrm{f}'(x)\dfrac{1}{\sqrt{x+6}}+\dfrac{6}{x^2}$ and that $\mathrm{f}(3) = 1$.

Find $\mathrm{f}(x)$. [5]

$f'(x) = (x+6)^{-\frac{1}{2}} + 6x^{-2}$

$\therefore f(x) = 2(x+6)^{\frac{1}{2}} - 6x^{-1} + (C)$ M1

A1 A1

When $x = 3$: $1 = 2(9)^{\frac{1}{2}} - 6(3)^{-1} + C$ M1

$1 = 4 + C$

$C = -3$ A1

$(\therefore f(x) = 2(x+6)^{\frac{1}{2}} - 6x^{-1} - 3)$

Examiner comment

The first method mark is awarded for a clear attempt at integration – usually an increase in the power of each term is sufficient.

Question 5

5 (a) The curve $y = x^2 + 3x + 4$ is translated by $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$.

Find and simplify the equation of the translated curve. [2]

$y = (x - 2)^2 + 3(x - 2) + 4$ M1

$y = x^2 - 4x + 4 + 3x - 6 + 4$

$y = x^2 - x + 2$ A1

Examiner comment

No specific comments here.

(b) The graph of $y = f(x)$ is transformed to the graph of $y = 3f(-x)$.

Describe fully the two single transformations which have been combined to give the resulting transformation. [3]

A reflection in the y axis. B1

A stretch in the y direction, with a scale factor of 3. B1 B1

Examiner comment

No specific comments here.

Question 6

6 **(a)** Find the coefficients of x^2 and x^3 in the expansion of $(2-x)^6$. [3]

$$(2-x)^6 = 2^6 + \binom{6}{1}2^5(-x) + \binom{6}{2}2^4(-x)^2 + \binom{6}{3}2^3(-x)^3$$

Coefficient of $x^2 = \binom{6}{2} 2^4 = 240$ B1

Coefficient of $x^3 = \binom{6}{3} 2^3(-1)^3 = -160$ B1 B1

Examiner comment

The candidate should be aware of the structure of this expansion, although its inclusion is not required for full marks.

For the coefficient of x^3, one of the B marks can be awarded if the candidate has got the wrong sign (i.e. + instead of −160).

(b) Hence find the coefficient of x^3 in the expansion of $(3x+1)(2-x)^6$ [2]

$(3x+1)(\ldots 240x^2 - 160x^3 \ldots)$

Coefficient of x^3 : $3 \times 240 + 1 \times (-160)$ M1

$= 560$ A1

Examiner comment

The first line of working shows that the candidate knows where the solution comes from, but it is not required for full marks to be awarded. The method mark is awarded for 3 × *their* 240 + 1 × *their* (−160) and the A mark for a *correct* follow through.

Question 7

7 (a) Show that the equation $1 + \sin x \tan x = 5 \cos x$ can be expressed as

$$6 \cos^2 x - \cos x - 1 = 0.$$

[3]

$1 + \sin x \tan x = 5 \cos x$

$1 + \sin x \dfrac{\sin x}{\cos x} = 5 \cos x$ M1

$\cos x + \sin^2 x = 5 \cos^2 x$

$\cos x + (1 - \cos^2 x) = 5 \cos^2 x$ M1

$0 = 6 \cos^2 x - \cos x - 1$ A1

or $6 \cos^2 x - \cos x - 1 = 0$

Examiner comment

Method marks are awarded for using the correct replacement for tan x, and for using the correct identity ($\sin^2 x = 1 - \cos^2 x$) in an appropriate place.

(b) Hence solve the equation $1 + \sin x \tan x = 5 \cos x$ for $0° \leqslant x \leqslant 180°$. [3]

$6 \cos^2 x - \cos x - 1 = 0$

$(3\cos x + 1)(2\cos x - 1) = 0$ M1

$\therefore \cos x = -\dfrac{1}{3}$ or $\dfrac{1}{2}$

$x = 60°$ or $109.5°$ (1 dp) A1 A1

Examiner comment

A correct method of solving a quadratic equation must be seen – without the method the two A1 marks cannot be awarded.

Question 8

8 A curve has equation $y = \dfrac{12}{3 - 2x}$.

(a) Find $\dfrac{dy}{dx}$. [2]

$y = 12(3 - 2x)^{-1}$

$\dfrac{dy}{dx} = -12(3 - 2x)^{-2}$ B1 $\times -2$ B1

$= 24(3 - 2x)^{-2}$

Examiner comment

Working is not required in this question for full marks to be awarded.

A point moves along this curve. As the point passes through A, the x-coordinate is increasing at a rate of 0.15 units per second and the y-coordinate is increasing at a rate of 0.4 units per second.

(b) Find the possible x-coordinates of A. [4]

$\frac{dx}{dt} = 0.15, \frac{dy}{dt} = 0.4.$

$\therefore \frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$

$= 0.4 \div 0.15$ M1

$= \frac{8}{3}$

$\therefore 24(3-2x)^{-2} = \frac{8}{3}$

$72 = 8(3-2x)^2$

$9 = (3-2x)^2$ M1

$\pm 3 = 3 - 2x$

Either $x = 0$ A1 or $3 - 2x = -3$

$x = 3$ A1

Examiner comment

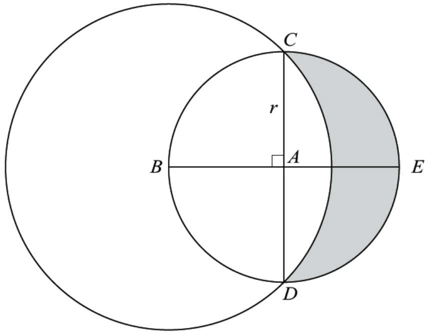
The first two lines show the expected thought process but full marks can be awarded without it being seen.

The first method mark is awarded when the candidate's algebraic expression for $\frac{dy}{dx}$ is equated to their numeric value obtained from the use of the chain rule, and a method for solving a quadratic equation is seen.

The second method mark is awarded for an attempt to equate *their* solution to part **(a)** to *their* numeric value of $\frac{dy}{dx}$ found above, provided this leads to them solving a quadratic equation. There is no follow through marks awarded here.

Question 9

9



The diagram shows a circle with centre *A* and radius *r*. Diameters *CAD* and *BAE* are perpendicular to each other. A larger circle has centre *B* and passes through *C* and *D*.

(a) Show that the radius of the larger circle is $r\sqrt{2}$. [1]

BC is the radius of the larger circle.

$BC^2 = AB^2 + AC^2$

$\therefore BC^2 = r^2 + r^2$

$= 2r^2$

$\therefore BC = r\sqrt{2}$ B1

Examiner comment

A valid method must be seen before the mark can be awarded – it does not have to be the method shown here.

(b) Find the area of the shaded region in terms of r. [6]

Shaded area = $\frac{1}{2}$ Area of small circle – Area of the minor segment of the large circle on the chord ACD.

$C\hat{B}D = \frac{\pi}{2}$

$\therefore$ Segment area = Sector area – triangle area

$$= \frac{1}{2}(r\sqrt{2})^2 \frac{\pi}{2} - \frac{1}{2}(r\sqrt{2})^2 \sin\frac{\pi}{2}$$ M1 M1

$$= \frac{1}{2}r^2\pi - r^2$$ A1

Shaded area $= \frac{1}{2}\pi r^2$ B1 $-\left(\frac{1}{2}r^2\pi - r^2\right)$ M1

$= r^2$ A1

Examiner comment

No specific comments here.

Question 10

10 The circle $x^2 + y^2 + 4x - 2y - 20 = 0$ has centre C and passes through points A and B.

(a) State the coordinates of C. [1]

From the given equation $(x+2)^2 + (y-1)^2 = r^2$

$\therefore C$ is $(-2, 1)$ B1

Examiner comment

This is the form of the equation of a circle that the candidate is expected to know, but the mark is awarded for stating the coordinates of *C*.

It is given that the midpoint, D, of AB has coordinates $\left(1\frac{1}{2}, 1\frac{1}{2}\right)$.

(b) Find the equation of AB, giving your answer in the form $y = mx + c$. [4]

Gradient $CD = \dfrac{1 - 1\frac{1}{2}}{-2 - 1\frac{1}{2}} = \dfrac{-\frac{1}{2}}{-3\frac{1}{2}} = \dfrac{1}{7}$ B1 $\therefore$ gradient $AB = -7$ M1

Equation of AB : $y - 1\frac{1}{2} = -7\left(x - 1\frac{1}{2}\right)$ M1

Giving $y = -7x + 12$ A1

Examiner comment

The form of the line equation used here for the equation of AB is awarded M1. If $y = mx + c$ is used directly, the mark is not awarded until the attempt at finding c is completed.

(c) Find, by calculation, the x-coordinates of A and B. [3]

Using the equations of the line and the circle:

$$x^2 + (-7x + 12)^2 + 4x - 2(-7x + 12) - 20 = 0$$ M1

$$x^2 + 49x^2 - 168x + 144 + 4x + 14x - 24 - 20 = 0$$

$$50x^2 - 150x + 100 = 0$$ A1

$$x^2 - 3x + 2 = 0$$

$$(x - 1)(x - 2) = 0$$

$$\therefore x = 1 \text{ or } 2$$ A1

Examiner comment

There must be clear evidence of a method of solution of a quadratic equation for the final A1 to be awarded.

Question 11

11 The function f is defined, for $x \in \mathbb{R}$, by $f : x \mapsto x^2 + ax + b$, where a and b are constants.

(a) It is given that $a = 6$ and $b = -8$.

Find the range of f. [3]

$f(x) = x^2 + 6x - 8$

$= (x+3)^2 - 17$ B1 B1

$\therefore f(x) \geqslant -17$ B1

Examiner comment

The final B mark is a follow through from the candidate's value of the *y*-coordinate (either expressed explicitly, or seen in the completed square form).

It is given instead that $a = 5$ and that the roots of the equation $f(x) = 0$ are k and $-2k$, where k is a constant.

(b) Find the values of b and k. [3]

$f(x) = x^2 + 5x + b$

$\therefore (x-k)(x+2k) = x^2 + 5x + b$ M1

$\therefore x^2 + kx - 2k^2 = x^2 + 5x + b$

Comparing coefficients: $k = 5$ A1 and $-2k^2 = b$

$\therefore b = -50$ A1 and $k = 5$.

Examiner comment

No specific comment here.

(c) Show that if the equation f$(x + a) = a$ has no real roots then $a^2 < 4(b - a)$. [3]

$$f(x+a) = (x+a)^2 + a(x+a) + b$$ M1

$$= x^2 + 3ax + 2a^2 + b$$

When $f(x + a) = a$:

$$x^2 + 3ax + 2a^2 + b - a = 0$$

For no real roots "$b^2 - 4ac$" < 0

$$\therefore 9a^2 - 4 \times 1 \times (2a^2 + b - a) < 0$$ DM1

$$9a^2 - 8a^2 - 4b + 4a < 0$$

$$a^2 < 4b - 4a$$

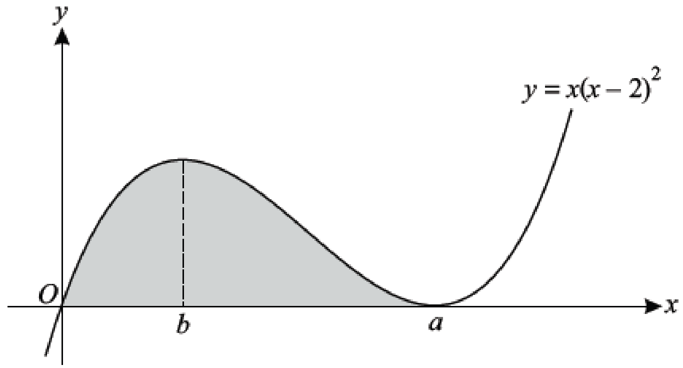
$$a^2 < 4(b - a)$$ A1

Examiner comment

No specific comment here.

Question 12

12



The diagram shows the curve with equation $y = x(x-2)^2$. The minimum point on the curve has coordinates $(a, 0)$ and the x-coordinate of the maximum point is b, where a and b are constants.

(a) State the value of a. [1]

The repeated factor $(x - 2)$ in the cubic equation means that the curve touches the x axis at $x = 2$.

Therefore $a = 2$. B1

Examiner comment

The mark is awarded for stating $a = 2$.

(b) Calculate the value of b. [4]

$y = x(x-2)^2 = x(x^2 - 4x + 4)$

$= x^3 - 4x^2 + 4x$ B1

$\frac{dy}{dx} = 3x^2 - 8x + 4$
B1 B1

At turning points $\frac{dy}{dx} = 0$

$\therefore 0 = 3x^2 - 8x + 4$

$0 = (x-2)(3x-2)$

$(x-2) = 0$ gives a

Therefore b comes from $(3x-2) = 0$

$3x = 2$

$x = \frac{2}{3}$ B1

Examiner comment

The mark is awarded for stating $x = \frac{2}{3}$. This mark is dependent on clear evidence of a method of solution of a quadratic equation. The two B marks for the differentiation are follow-through marks from the candidate's expansion of $y = x(x-2)^2$.

(c) Find the area of the shaded region. [4]

$$A = \int_0^2 x^3 - 4x^2 + 4x \, dx$$

$$= \left[\frac{x^4}{4} - \frac{4x^3}{3} + 2x^2\right]_0^2$$ B1 B1

$$= \left(4 - \frac{32}{3} + 8\right) - 0$$ M1

$$= \frac{4}{3}$$ A1

Examiner comment

The method mark here is **not** implied by a correct answer; this use of limits must be seen.

(d) The gradient, $\frac{dy}{dx}$, of the curve has a minimum value m.

Calculate the value of m. [4]

The minimum value of $\frac{dy}{dx}$ occurs when $\frac{d^2y}{dx^2} = 0$.

$$\frac{d^2y}{dx^2} = 6x - 8$$

$$0 = 6x - 8$$

$$x = \frac{4}{3}$$ M1 A1

When $x = \frac{4}{3}$, $\frac{dy}{dx} = 3\left(\frac{4}{3}\right)^2 - 8\left(\frac{4}{3}\right) + 4$ DM1

$$= -\frac{4}{3}$$ A1

Examiner comment

This is the thought process expected of the candidate.

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