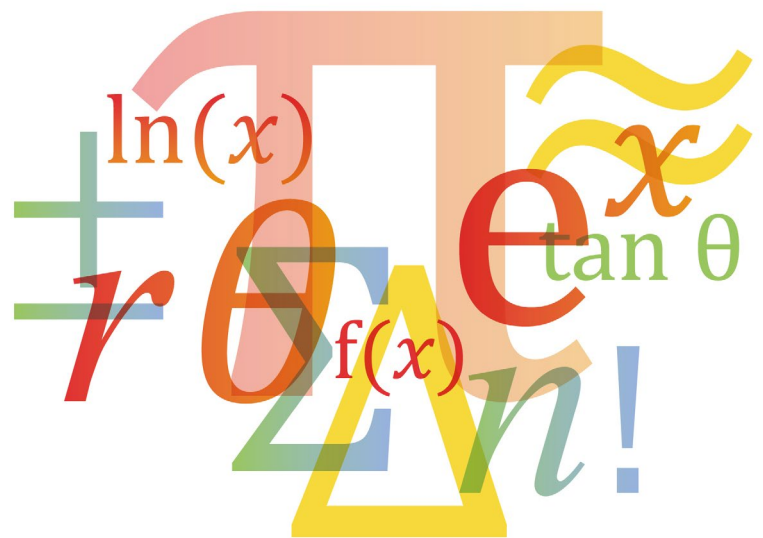




Learner Guide

Cambridge International AS & A Level Mathematics 9709

For examination from 2020



In order to help us develop the highest quality resources, we are undertaking a continuous programme of review; not only to measure the success of our resources but also to highlight areas for improvement and to identify new development needs.

We invite you to complete our survey by visiting the website below. Your comments on the quality and relevance of our resources are very important to us.

www.surveymonkey.co.uk/r/GL6ZNJB

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About this guide

This guide explains what you need to know about your Cambridge International AS & A Level Mathematics course and examinations.

This guide will help you to:

- ✓ understand what skills you should develop by taking this Cambridge International AS & A Level course
- ✓ understand how you will be assessed
- ✓ understand what we are looking for in the answers you write
- ✓ plan your revision programme
- ✓ revise, by providing revision tips and an interactive revision checklist (Section 6).

Following a Cambridge International AS & A level programme will help you to develop abilities that universities value highly, including a deep understanding of your subject; higher order thinking skills (analysis, critical thinking, problem solving); presenting ordered and coherent arguments; and independent learning and research.

Studying Cambridge International AS & A Level **Mathematics** will help you to develop a set of transferable skills, including the ability to work with mathematical information; think logically and independently; consider accuracy; model situations mathematically; analyse results and reflect on findings.

Section 1: Syllabus content - what you need to know

This section gives you an outline of the syllabus content for this course. There are six components that can be combined in specific ways (see Section 3). Talk to your teacher to make sure you know which components you will be taking.

Make sure you always check the latest syllabus, which is available from our [public website](#). This will also explain the different combinations of components you can take.

Prior knowledge

Knowledge of the content of the Cambridge IGCSE™ Mathematics 0580 (Extended curriculum), or Cambridge International O Level (4024/4029), is assumed.

Key concepts

Key concepts are essential ideas that help you to develop a deep understanding of your subject and make links between different aspects of the course. The key concepts for Cambridge International AS & A Level Mathematics are:

Problem solving

Mathematics is fundamentally problem solving and representing systems and models in different ways. These include:

- Algebra: this is an essential tool which supports and expresses mathematical reasoning and provides a means to generalise across a number of contexts.
- Geometrical techniques: algebraic representations also describe a spatial relationship, which gives us a new way to understand a situation.
- Calculus: this is a fundamental element which describes change in dynamic situations and underlines the links between functions and graphs.
- Mechanical models: these explain and predict how particles and objects move or remain stable under the influence of forces.
- Statistical models: these are used to quantify and model aspects of the world around us. Probability theory predicts how chance events might proceed, and whether assumptions about chance are justified by evidence.

Communication

Mathematical proof and reasoning is expressed using algebra and notation so that others can follow each line of reasoning and confirm its completeness and accuracy. Mathematical notation is universal. Each solution is structured, but proof and problem solving also invite creative and original thinking.

Mathematical modelling

Mathematical modelling can be applied to many different situations and problems, leading to predictions and solutions. A variety of mathematical content areas and techniques may be required to create the model. Once the model has been created and applied, the results can be interpreted to give predictions and information about the real world.

Section 2: How you will be assessed

Cambridge International AS Level Mathematics makes up the first half of the Cambridge International A Level course in mathematics and provides a foundation for the study of mathematics at Cambridge International A Level.

About the examinations

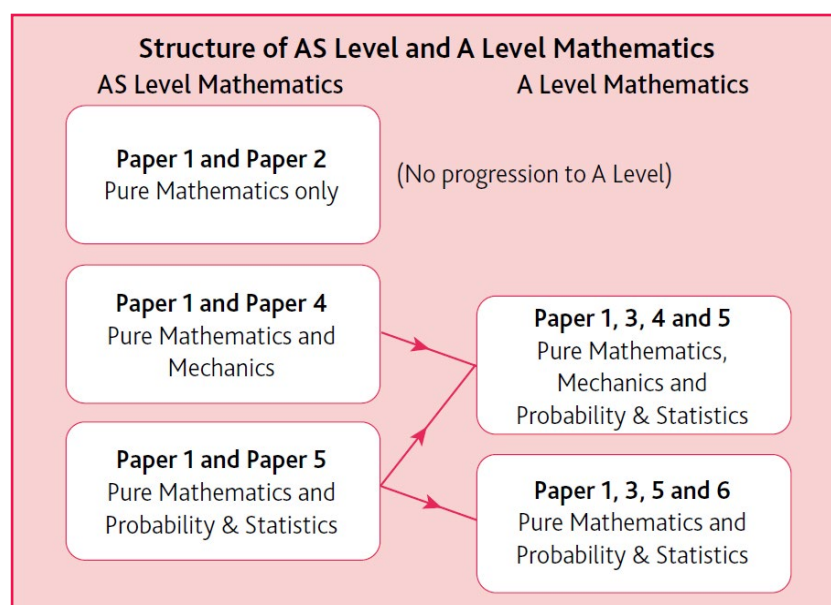
There are three different combinations of papers you can take to obtain an AS level Mathematics qualification:

- Papers 1 and 2 (this cannot lead to an A Level route)
- Papers 1 and 4
- Papers 1 and 5

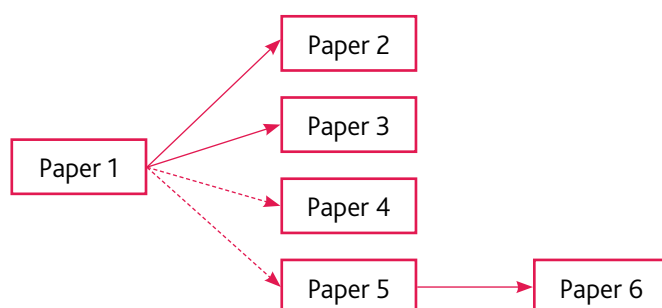
There are two different combinations of papers you can take to obtain an A Level Mathematics qualification:

- Papers 1, 3, 4 and 5
- Papers 1, 3, 5 and 6.

These are summarised in the diagram. Find out from your teacher which papers you will be taking.



Your knowledge will build as you progress through the course. Paper 1 Pure Mathematics 1 is the foundation for all other components. (Solid lines mean there is direct dependency of one paper on another; dashed lines mean that prior knowledge from previous paper is assumed.)



About the papers

The table gives you further information about the examination papers:

Component	Time and marks	Questions	Percentage of qualification
Paper 1 Pure Mathematics 1	1 hour 50 minutes (75 marks)	A written paper with 10 to 12 structured questions based on the subject content for Pure Mathematics 1. You must answer all questions.	60% of AS Level 30% of A Level
Paper 2 Pure Mathematics 2	1 hour 15 minutes (50 marks)	A written paper with 6 to 8 structured questions based on the subject content for Pure Mathematics 2. You must answer all questions.	40% of AS level (not offered as part of A Level)
Paper 3 Pure Mathematics 3	1 hour 50 minutes (75 marks)	A written paper with 9 to 11 structured questions based on the subject content for Pure Mathematics 3. You must answer all questions.	(not offered as part of AS Level; compulsory for A Level) 30% of A Level
Paper 4 Mechanics	1 hour 15 minutes (50 marks)	A written paper with 6 to 8 structured questions based on the subject content for Mechanics. You must answer all questions.	40% of AS level 20% of A Level
Paper 5 Probability and Statistics 1	1 hour 15 minutes (50 marks)	A written paper with 6 to 8 structured questions based on the subject content for Probability & Statistics 1. You must answer all questions.	40% of AS level 20% of A Level
Paper 6 Probability and Statistics 2	1 hour 15 minutes (50 marks)	A written paper with 6 to 8 structured questions based on the subject content for Probability & Statistics 2. You must answer all questions.	(not offered as part of AS Level) 20% of A Level

Section 3: What skills will be assessed

The examiners take account of the following skills areas (**assessment objectives**) in the examinations:

Assessment objectives (AO)	What does the AO mean?	
AO1 Knowledge and understanding <ul style="list-style-type: none"> Show understanding of relevant mathematical concepts, terminology and notation Recall accurately and use appropriate mathematical manipulative techniques 	Demonstrating that you understand what is required of a question. Remembering methods and notation. Showing your working and the methods you used will help you demonstrate your AO1 skills.	Paper 1 (55% of marks) Paper 2 (55% of marks) Paper 3 (45% of marks) Paper 4 (55% of marks) Paper 5 (55% of marks) Paper 6 (55% of marks)
AO2 Application and communication <ul style="list-style-type: none"> Recognise the appropriate mathematical procedure for a given situation Apply appropriate combinations of mathematical skills and techniques in solving problems Present relevant mathematical work, and communicate corresponding conclusions, in a clear and logical way 	Deciding which methods to use when solving a problem, and how/when to apply more than one method. You need to write your solution clearly and logically so that someone else can understand it. Showing your working and the methods you used will help you demonstrate your AO2 skills.	Paper 1 (45% of marks) Paper 2 (45% of marks) Paper 3 (55% of marks) Paper 4 (45% of marks) Paper 5 (45% of marks) Paper 6 (45% of marks)

It is important that you know the different weightings (%) of the assessment objectives, as this affects how the examiner will assess your work.

Assessment objectives as a percentage of each qualification

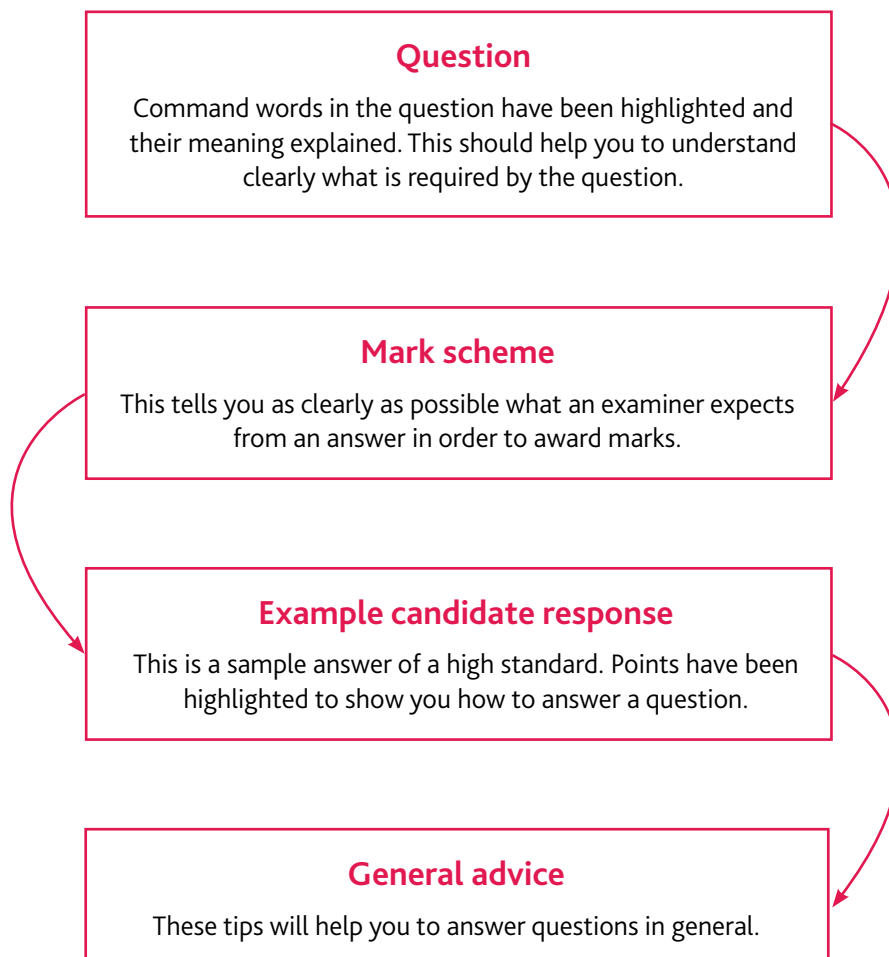
Assessment objective	Weighting at AS Level %	Weighting at A Level %
AO1 Knowledge and understanding	55	52
AO2 Application and communication	45	48
Total	100	100

Section 4: Example candidate response

This section takes you through an example question and candidate response. It will help you to see how to identify the command words within questions and to understand what is required in your response. Understanding the questions will help you to know what you need to do with your knowledge. For example, you might need to state something, calculate something, find something or show something.

All information and advice in this section is specific to the example question and response being demonstrated. It should give you an idea of how your responses might be viewed by an examiner but it is not a list of what to do in all questions. In your own examination, you will need to pay careful attention to what each question is asking you to do.

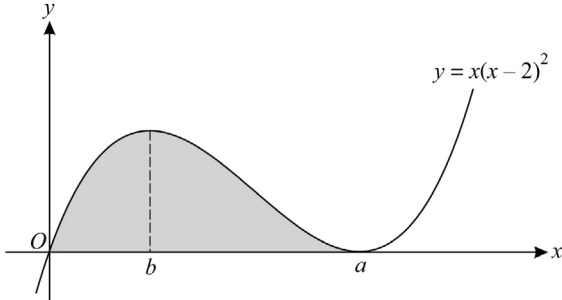
This section is separated as follows:



Question

The question used in this example is from Paper 1 Pure Mathematics 1. The answer spaces have been removed from the layout below so that it all fits on one page.

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The diagram shows the curve with equation $y = x(x - 2)^2$. The minimum point on the curve has coordinates $(a, 0)$ and the x -coordinate of the maximum point is b , where a and b are constants.

(a) State the value of a . [1]

State – the examiner will be expecting you to write down a value of a without any working.

(b) Calculate the value of b . [4]

Calculate – the examiner will be expecting you to apply differentiation methods to work out b (the x -coordinate of the maximum) in part (b) and m (the minimum gradient of the curve) in part (d).

(c) Find the area of the shaded region. [4]

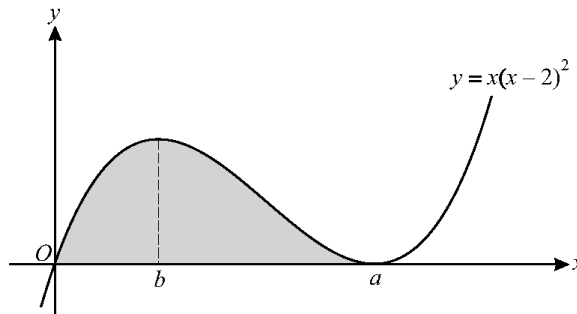
Find – is a key instruction word. The examiner will be expecting you to apply integration methods to find the area of the shaded region in part (c).

(d) The gradient, $\frac{dy}{dx}$, of the curve has a minimum value m .
Calculate the value of m . [4]

Example candidate response

This model answer is awarded full marks, i.e. 13 out of 13. The Examiner comments are in the boxes.

12



The diagram shows the curve with equation $y = x(x - 2)^2$. The minimum point on the curve has coordinates $(a, 0)$ and the x -coordinate of the maximum point is b , where a and b are constants.

- (a) State the value of a . [1]

$a = 2$

State is a clue that not much work is needed and you can just write down the answer. You know that $y = 0$ at this point and the function is already factorised.

- (b) Calculate the value of b . [4]

$$y = x(x-2)^2 = x(x^2 - 4x + 4)$$

$$= x^3 - 4x^2 + 4x$$

Expand expression $(x - 2)^2$ to obtain the correct cubic.

$$\frac{dy}{dx} = 3x^2 - 8x + 4$$

Differentiate all of the terms.

$$\frac{dy}{dx} = 0 \Rightarrow 3x^2 - 8x + 4 = 0$$

$$(3x - 2)(x - 2) = 0$$

$$x = \frac{2}{3}, 2$$

$$a = 2 \text{ so } b = \frac{2}{3}$$

Set $\frac{dy}{dx} = 0$ and solve an equation to find b , the x -coordinate of the maximum. You need to show how you solved the equation to find the two values, then write down b . It is not enough to solve the quadratic equation on your calculator and write down the answers, although you can use your calculator to check once you have done the working on the Paper.

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(c) Find the area of the shaded region. [4]

$$\text{Area} = \int_0^2 x^3 - 4x^2 + 4x \, dx$$

$$= \left[\frac{x^4}{4} - \frac{4x^3}{3} + 2x^2 \right]_0^2$$

$$= \frac{16}{4} - \frac{4 \times 8}{3} + 8 - 0$$

$$= 12 - \frac{32}{3} = \frac{4}{3}$$

Integrate your expression for y , using correct notation.

Substitute in the limits, 0 and your value of a , to get the method mark. When you think about which limits to use, you will probably realise that your value for a is one of them. The other one is 0 and not b . Reading the question carefully and thinking about the graph will help you. If it helps, you could also label the graph with the values you have found.

Obtain the correct answer (you need to show the method for finding it).

(d) The gradient, $\frac{dy}{dx}$, of the curve has a minimum value m .

Calculate the value of m .

$$\frac{dy}{dx} = 3x^2 - 8x + 4$$

To see where $\frac{dy}{dx}$ is a maximum, find $\frac{d^2y}{dx^2}$ and write $\frac{d^2y}{dx^2} = 0$

$$\frac{d^2y}{dx^2} = 6x - 8 = 0 \Rightarrow x = \frac{4}{3}$$

Solve the equation to find x .

so the minimum value of $\frac{dy}{dx}$ occurs when $x = \frac{4}{3}$

Substitute this value into $\frac{dy}{dx}$.

$$\frac{dy}{dx} = 3\left(\frac{4}{3}\right)^2 - 8\left(\frac{4}{3}\right) + 4 = \frac{16}{3} - \frac{32}{3} + 4$$

Obtain the correct value of m .

$$\therefore m = -\frac{4}{3}$$

Some learners might find part (d) a little unfamiliar. Don't be tempted to repeat what you've already done or substitute values into the 2nd derivative. It is better to stop and think about what you need to know. You are trying to analyse the gradient to see where it is a minimum. Just as the 1st derivative tells you about a minimum in the original function, y , so the 2nd derivative tells you about a minimum in the 1st derivative (it's the gradient of the gradient function).
So one way to answer this is to put $\frac{d^2y}{dx^2} = 0$, solve this for x and then use that value of x to find $\frac{dy}{dx}$ (or m). This is the method shown in the mark scheme. Another valid way of answering this question is to complete the square on $\frac{dy}{dx}$. This gives you $3\left(x - \frac{4}{3}\right)^2 - \frac{4}{3}$. You can see that the minimum value (m) would be $-\frac{4}{3}$ because $\left(x - \frac{4}{3}\right)^2$ is always ≥ 0 . There are other valid ways of answering the question too. All of these would allow you to gain M marks and, if the answers are correct, A marks too.

General advice

It is always a good idea to read the question carefully, noticing the command words and key instructions (in this case 'State', 'Find' and 'Calculate'). You may want to underline them to help you think what they mean. Many candidates jump straight into writing their working only to realise too late that they've used the wrong method. Read the question first and pause to think about what you need to find before you start doing any working – this will help you to choose an efficient method so you don't waste time in the examination. Don't forget that your working is part of your solution and you can gain method marks even if you don't get as far as a correct answer. In the example question, there are several FT (follow through) marks as well. This means that if you can go on to use *your* values correctly, even if they are wrong, you can still get subsequent method marks.

When answering a question based on a graph, such as the example question, it is often helpful to add to the graph any values you find, or information from the question. This will help you to think about what methods you can use to answer the question. If the question doesn't provide a graph or diagram, it is often useful for you to sketch your own.

Using correct notation in your working will help you to think clearly as well as making it easier for someone else to understand what you have done.

Make sure you are clear if you need to differentiate or integrate. It is surprisingly common for learners to get confused when a question requires both methods.

If you have had a good attempt at a question and still not managed to finish it, it is best to move on to another question and come back to it later. This will help you to make good use of the time you have available.

Allow a few minutes at the end of the examination to check your work. This will help you to spot errors in your arithmetic that could lose you marks.

Section 5: Revision

This advice will help you revise and prepare for the examinations. It is divided into general advice and specific advice for each of the papers.

Use the tick boxes to keep a record of what you have done, what you plan to do or what you understand.

General advice

Before the examination

- Find out when the examinations are and plan your revision so you have enough time for each topic. A revision timetable will help you.
- Find out how long each Paper is and how many questions you have to answer.
- Know the meaning of the command words used in questions and how to apply them to the information given. Highlight the command words in past papers and check what they mean. There is a list on p11 of this guide.
- Make revision notes; try different styles of notes. See the Learner Guide: Planning, Reflection and Revision (www.cambridgeinternational.org/images/371937-learner-guide-planning-reflection-and-revision.pdf) which has ideas about note-taking. Discover what works best for you.
- Work for short periods then have a break. Revise small sections of the syllabus at a time.
- Build your confidence by practising questions on each of the topics.
- Make sure you practise lots of past examination questions so that you are familiar with the format of the examination papers. You could time yourself when doing a paper so that you know how quickly you need to work in the real examination.
- Look at mark schemes to help you to understand how the marks are awarded for each question.
- Make sure you are familiar with the mathematical notation that you need for this syllabus. Your teacher will be able to advise you on what is expected.
- Check which formulae are in the formula booklet available in the examination, and which ones you need to learn.

During the examination

- Read the instructions carefully and answer **all** the questions.
- Check the number of marks for each question or part question. This helps you to judge how long you should spend on the response. You don't want to spend too long on some questions and then run out of time at the end.
- Do not leave out questions or parts of questions. Remember, no answer means no mark.
- You do not have to answer the questions in the order they are printed in the answer booklet. You may be able to do a later question more easily then come back to an earlier one for another try.
- Read each question very carefully. Misreading a question can cost you marks:
 - Identify the command words – you could underline or highlight them.
 - Identify the other key words and perhaps underline them too.
 - Try to put the question into your own words to understand what it is really asking.
- Read all parts of a question before starting your answer. Think carefully about what is needed for each part. You will not need to repeat material.
- Look very carefully at the information you are given.
 - For graphs, read the title, key, axes, etc. to find out exactly what they show.
 - For diagrams, look at any angles and lengths.
 - Try using coloured pencils or pens to pick out anything that the question asks you about.

- Answer the question.** This is very important!
 - Use your knowledge and understanding.
 - Do not just try all the methods you know. Only use the ones you need to answer the question.
- Make sure that you have answered everything that a question asks. Sometimes one sentence asks two things, e.g. 'Show that ... and hence find ...'. It is easy to concentrate on the first request and forget about the second one.
- Always show your working. Marks are usually awarded for using correct steps in the method even if you make a mistake somewhere.
- Do not cross out any working until you have replaced it by trying again. Even if you know it's not correct you may still be able to get method marks. If you have made two or more attempts, make sure you cross out all except the one you want marked.
- Use mathematical terms in your answers when possible.
- Annotated diagrams and graphs can help you, and can be used to support your answer. Use them whenever possible but do not repeat the information in words.
- Make sure all your numbers are clear, for example make sure your '1' doesn't look like a '7'.
- If you need to change a word or a number, or even a sign (+ to – for example), it is better to cross out your work and rewrite it. Do not try to write over the top of your previous work as it will be difficult to read and you may not get the marks.
- Do not write your answers in two columns in the examination. It is difficult for the examiners to read and follow your working.

Advice for all Papers

- Give numerical answers correct to 3 significant figures (3SF) in questions where no accuracy is specified, except for angles in degrees, which need to be to 1 decimal place (1 d.p.).
- Make sure you know the difference between 3 significant figures and 3 decimal places, e.g. 0.03456 is 0.0346 (3 SF) but is 0.035 (3 d.p.). You would not get the mark for 0.035 as it is not accurate enough.
- For your answers to be accurate to 3SF, you will have to work with at least 4SF throughout the question. For a calculation with several stages, it is usually best to use all the figures in your calculator. However, you do not need to write all these figures down in your working but you should write down each to at least 4SF before rounding your **final** answer to 3SF.
- If a question specifies how accurate your answer needs to be, you **must** give your final answer to that degree of accuracy. In questions where the accuracy is not specified, however, you will not be penalised if you give answers that are **more** accurate than 3SF.
- Some questions ask for answers in exact form. In these questions you must **not** use your calculator to evaluate answers and you must show the steps in your working. Exact answers may include fractions or square roots and you should simplify them as far as possible.
- You are expected to use a scientific calculator in all your examination papers. You are **not** allowed to use computers, graphical calculators and calculators capable of symbolic algebraic manipulation or symbolic differentiation or integration.
- Check that your calculator is in degree mode for questions that have angles in degrees and in radian mode for questions that have angles in radians.
- In questions where you have to show an answer that is given, it is particularly important to show all your working. To gain the marks, you need to convince the examiner that you understand all the steps in getting to the answer.
- Often the 'given' answer from a 'show that' question is needed in the next part of the question. This means that even if you couldn't show how to obtain it, you can still carry on with other parts of the question. In later parts that rely on the 'given' answer, you should **always** use the 'given' answer exactly as it is stated in the question, even if you have obtained a different answer yourself.
- There are no marks available for just stating a method or a formula. You have to apply the method to the particular question, or use the formula by substituting values in.

Advice for Paper 1, 2 and 3

- Make sure you know all the formulae that you need (even ones from Cambridge IGCSE or O Level). If you use an incorrect formula you will score no marks.
- Check to see if your answer is required in exact form. In a trigonometry question you will need to use exact values of $\sin 60^\circ$, for example, to obtain an exact answer. Make sure you know the exact values of \sin , \cos and \tan of 30° , 45° and 60° as they are not provided in the examination.
- Calculus questions (Paper 2 and 3) involving trigonometric functions use values in radians, so make sure your calculator is in the correct mode if you need to use it.

Paper 4 advice

- When you need a numerical value for 'g', use 10 ms^{-2} (unless the question states otherwise).
- Always draw clear force diagrams, whether the question asks for them or not. This will help you to solve problems.
- Make sure you are familiar with common words such as 'initial', 'resultant', 'smooth', 'rough', 'light' and 'inextensible'. Also make sure that you know the difference between 'mass' and 'weight'.
- Go through some past examination questions and highlight common words and phrases. Learn what they mean and make sure you can recognise them.
- Make sure you know the algebraic methods from Paper 1 Pure Mathematics 1.
- Make sure you know these trigonometrical results:
 $\sin(90^\circ - \theta) = \cos \theta$, $\cos(90^\circ - \theta) = \sin \theta$, $\tan \theta = \frac{\sin \theta}{\cos \theta}$, $\sin^2 \theta + \cos^2 \theta = 1$
- When a question mentions bodies in a 'realistic' context, you should still treat them as particles.
- Vector notation will not be used.

Advice for Paper 5 and 6

- You can give probabilities as fractions, decimals or percentages in your answer.
- Always look to see if your answer makes sense. For example, if you calculated a probability as 1.2, you would know you had made a mistake and should check your solution.
- Sometimes you might be asked to give an answer 'in the context of the question'. This means that your answer must use information about the situation described in the question. For example, don't just write '*The events must be independent*' (which could apply in any situation). If the question is about scores on a dice for instance, you could write 'The scores when the dice is rolled must be independent'. If the question is about the time taken by people to carry out a task, you could write '*The times taken by the people must be independent of each other*'.
- When you are answering a question about a normal distribution, it is useful to draw a diagram; this can help you to spot errors. For example, if you are finding a probability you will be able to see from a diagram if you would expect the answer to be greater or less than 0.5.

Paper 6 advice

- When you are carrying out a hypothesis test, you should always state the conclusion in the context of the question. Don't state conclusions in a way that implies that a hypothesis test has proved something. For example, it is better to write '*There is evidence that the mean weight of the fruit has increased*' instead of '*The test shows that the mean weight of the fruit has increased*'.
- When you are calculating a confidence interval, it may not always be sensible to apply the usual 3 SF rule about accuracy. For example, if an interval for a population mean works out as (99.974, 100.316), it doesn't make sense to round both figures to 100. In this case, it is better to give 2 d.p. or 3 d.p. so that the width of the interval is more accurate.

Revision checklists

In the next part of this guide we have provided some revision checklists. These include information from the syllabus that you should revise. They do not contain all the detailed knowledge you need to know, just an overview. For more detail see the syllabus and talk to your teacher.

The table headings are explained below:

Topic	You should be able to	R	A	G	Comments
Here is a list of the topics you need to cover and work through.	This is the key content and understanding you need.	<p>You can use the tick boxes to show when you have revised an item and how confident you feel about it.</p> <p>R = RED means you are really unsure and lack confidence; you might want to focus your revision here and possibly talk to your teacher for help.</p> <p>A = AMBER means you are reasonably confident but need some extra practice.</p> <p>G = GREEN means you are very confident.</p> <p>As your revision progresses, you can concentrate on the RED and AMBER items in order to turn them into GREEN items. You might find it helpful to highlight each topic in red, orange or green to help you prioritise.</p>			<p>You can use the 'Comments' column to:</p> <ul style="list-style-type: none"> • add more information about the details for each point • add formulae or notes • include a reference to a useful resource • highlight areas of difficulty or things that you need to talk to your teacher about or look up in a textbook.

Note: the tables below cannot contain absolutely everything you need to know, but it does use examples wherever it can.

Paper 1 Pure Mathematics 1

Topic	You should be able to	R	A	G	Comments	
Quadratics	carry out the process of completing the square for a quadratic polynomial $ax^2 + bx + c$ and use a completed square form e.g. to locate the vertex of the graph $y = ax^2 + bx + c$, or to sketch the graph	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>		
	find the discriminant of a quadratic polynomial $ax^2 + bx + c$ and use the discriminant e.g. to determine the number of real roots of the equation $ax^2 + bx + c = 0$. Knowledge of the term 'repeated root' is included	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>		
	solve quadratic equations, and quadratic inequalities, in one unknown By factorising, completing the square and using the formula	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>		
	solve by substitution a pair of simultaneous equations of which one is linear and one is quadratic e.g. $x + y + 1 = 0$ and $x^2 + y^2 = 25$, $2x + 3y = 7$ and $3x^2 = 4 + 4xy$.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>		
	recognise and solve equations in x which are quadratic in some function of x . e.g. $x^4 - 5x^2 + 4 = 0$, $6x + \sqrt{x} - 1 = 0$, $\tan^2 x = 1 + \tan x$.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>		
	Functions	understand the terms function, domain, range, one-one function, inverse function and composition of functions	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
		identify the range of a given function in simple cases, and find the composition of two given functions Including e.g. range of $f: x \mapsto \frac{1}{x}$ for $x \geq 1$ and range of $g: x \mapsto x^2 + 1$ for $x \in \mathbb{R}$. Including the condition that a composite function gf can only be formed when the range of f is within the domain of g .	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
determine whether or not a given function is one-one, and find the inverse of a one-one function in simple cases Including e.g. finding the inverse of $h: x \mapsto (2x + 3)^2 - 4$ for $x < -\frac{3}{2}$		<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>		
illustrate in graphical terms the relation between a one-one function and its inverse Indication of the mirror line $y = x$ will be expected in sketches		<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>		

Topic	You should be able to	R	A	G	Comments
	understand and use the transformations of the graph of $y = f(x)$ given by $y = f(x) + a$, $y = f(x + a)$, $y = af(x)$, $y = f(ax)$ and simple combinations of these Use of the terms 'translation', 'reflection' and 'stretch' in describing transformations is included. Questions may involve algebraic or trigonometric functions, or other graphs with given features.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
Coordinate geometry	find the equation of a straight line given sufficient information e.g. given two points, or one point and the gradient	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	interpret and use any of the forms, $y = mx + c$, $y - y_1 = m(x - x_1)$, $ax + by + c = 0$, in solving problems Problems may involve calculations of distances, gradients, mid-points, points of intersection and use of the relationship between the gradients of parallel and perpendicular lines.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	understand that the equation $(x - a)^2 + (y - b)^2 = r^2$ represents the circle with centre (a, b) and radius r Including use of the expanded form $x^2 + y^2 + 2gx + 2fy + c = 0$.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	use algebraic methods to solve problems involving lines and circles Questions may require use of elementary geometrical properties of circles, e.g. tangent perpendicular to radius, angle in a semicircle, symmetry. Implicit differentiation is not included.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	understand the relationship between a graph and its associated algebraic equation, and use the relationship between points of intersection of graphs and solutions of equations e.g. to determine the set of values of k for which the line $y = x + k$ intersects, touches or does not meet a quadratic curve	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	understand the definition of a radian, and use the relationship between radians and degrees	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	use the formulae $s = r\theta$ and $A = \frac{1}{2}r^2\theta$ in solving problems concerning the arc length and sector area of a circle Problems may involve calculation of lengths and angles in triangles and areas of triangles.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	

Topic	You should be able to	R	A	G	Comments
Trigonometry	sketch and use graphs of the sine, cosine and tangent functions (for angles of any size, and using either degrees or radians)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	Including e.g. $y = 3 \sin x$, $y = 1 - \cos 2x$, $y = \tan \left(x + \frac{1}{4} \pi\right)$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	use the exact values of the sine, cosine and tangent of 30° , 45° , 60° , and related angles	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	e.g. $\cos 150^\circ = -\frac{1}{2} \sqrt{3}$, $\sin \frac{3}{4} \pi = \frac{1}{\sqrt{2}}$.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	use the notations $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$ to denote the principal values of the inverse trigonometric relations	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	No specialised knowledge of these functions is required, but understanding of them as examples of inverse functions is expected	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	use the identities $\frac{\sin \theta}{\cos \theta} \equiv \theta$ and $\sin^2 \theta + \cos^2 \theta \equiv 1$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	e.g. in proving identities, simplifying expressions and solving equations	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
Series	find all the solutions of simple trigonometrical equations lying in a specified interval (general forms of solution are not included)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	e.g. solve $3 \sin 2x + 1 = 0$ for $-\pi < x < \pi$, $3 \sin^2 \theta - 5 \cos \theta - 1 = 0$ for $0^\circ \leq \theta \leq 360^\circ$.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	use the expansion of $(a + b)^n$, where n is a positive integer	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	Knowledge of the greatest term and properties of the coefficients are not required, but the notations $\binom{n}{r}$ and $n!$ are included	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	recognise arithmetic and geometric progressions	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
use the formulae for the n^{th} term and for the sum of the first n terms to solve problems involving arithmetic or geometric progressions	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>		
	Problems may involve more than one progression. Knowledge that numbers a , b , c are 'in arithmetic progression' if $2b = a + c$ (or equivalent) and are 'in geometric progression' if $b^2 = ac$ (or equivalent).	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
use the condition for the convergence of a geometric progression, and the formula for the sum to infinity of a convergent geometric progression.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>		

Topic	You should be able to	R	A	G	Comments
Differentiation	understand the gradient of a curve at a point as the limit of the gradients of a suitable sequence of chords, and use the notations $f'(x)$, $f''(x)$, $\frac{dy}{dx}$, and $\frac{d^2y}{dx^2}$ for first and second derivatives	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	Only an informal understanding of the idea of a limit is expected. e.g. includes consideration of the gradient of the chord joining the points with x coordinates 2 and $(2 + h)$ on the curve $y = x^3$. Formal use of the general method of differentiation from first principles is not required.				
	use the derivative of x^n (for any rational n), together with constant multiples, sums, differences of functions, and of composite functions using the chain rule e.g. find $\frac{dy}{dx}$ given $y = \sqrt{2x^3 + 5}$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	apply differentiation to gradients, tangents and normals, increasing and decreasing functions and rates of change Connected rates of change are included; e.g. given the rate of increase of the radius of a circle, find the rate of increase of the area for a specific value of one of the variables	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	locate stationary points and determine their nature, and use information about stationary points in sketching graphs Use of the second derivative for identifying maxima and minima is included; alternatives may be used in questions where no method is specified. Knowledge of points of inflexion is not included	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
Integration	understand integration as the reverse process of differentiation, and integrate $(ax + b)^n$ (for any rational n except -1), together with constant multiples, sums and differences	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	e.g. $\int (2x^3 - 5x + 1) dx$, $\int \frac{1}{(2x + 3)^2} dx$				
	solve problems involving the evaluation of a constant of integration e.g. to find the equation of the curve through $(1, -2)$ for which $\frac{dy}{dx} = \sqrt{2x + 1}$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	evaluate definite integrals Simple cases of 'improper' integrals, such as $\int_0^1 x^{-\frac{1}{2}} dx$ and $\int_1^\infty x^{-2} dx$, are included.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	

Topic	You should be able to	R	A	G	Comments
	use definite integration to find <ul style="list-style-type: none"> the area of a region bounded by a curve and lines parallel to the axes, or between a curve and a line or between two curves, a volume of revolution about one of the axes A volume of revolution may involve a region not bounded by the axis of rotation, e.g. the region between $y = 9 - x^2$ and $y = 5$ rotated about the x-axis.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	

Paper 2 Pure Mathematics 2

Topic	You should be able to	R	A	G	Comments
Algebra	understand the meaning of $ x $, sketch the graph of $y = ax + b $ and use relations such as $ a = b \Leftrightarrow a^2 = b^2$ and $ x - a < b \Leftrightarrow a - b < x < a + b$ when solving equations and inequalities Graphs of $y = f(x) $ and $y = f(x)$ for non-linear functions f are not included. e.g. $ 3x - 2 = 2x + 7 $, $2x + 5 < x + 1 $	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	divide a polynomial, of degree not exceeding 4, by a linear or quadratic polynomial, and identify the quotient and remainder (which may be zero)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	use the factor theorem and the remainder theorem e.g. to find factors and remainders, solve polynomial equations or evaluate unknown coefficients. Factors of the form $(ax + b)$ in which the coefficient of x is not unity are included, and similarly for calculation of remainders	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
Logarithmic and exponential functions	understand the relationship between logarithms and indices, and use the laws of logarithms (excluding change of base)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	understand the definition and properties of e^x and $\ln x$, including their relationship as inverse functions and their graphs Knowledge of the graph of $y = e^{kx}$ for both positive and negative values of k is included.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	use logarithms to solve equations and inequalities in which the unknown appears in indices e.g. $2^x < 5$, $3 \times 2^{3x-1} < 5$, $3^{x+1} = 4 \cdot 2^{x-1}$.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	use logarithms to transform a given relationship to linear form, and hence determine unknown constants by considering the gradient and/or intercept e.g. $y = kx^n$, $y = k(a^x)$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	

Topic	You should be able to	R	A	G	Comments
Trigonometry	understand the relationship of the secant, cosecant and cotangent functions to cosine, sine and tangent, and use properties and graphs of all six trigonometric functions for angles of any magnitude	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	use trigonometrical identities for the simplification and exact evaluation of expressions and in the course of solving equations, and select an identity or identities appropriate to the context, showing familiarity in particular with the use of <ul style="list-style-type: none"> $\sec^2 \theta \equiv 1 + \tan^2 \theta$ and $\operatorname{cosec}^2 \theta \equiv 1 + \cot^2 \theta$ the expansions of $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$ the formulae for $\sin 2A$, $\cos 2A$ and $\tan 2A$ the expression of $a \sin \theta + b \cos \theta$ in the forms $R \sin(\theta \pm \alpha)$ and $R \cos(\theta \pm \alpha)$. 	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	e.g. simplifying $\cos(x - 30^\circ) - 3 \sin(x - 60^\circ)$. e.g. solving $\tan \theta + \cot \theta = 4$, $2 \sec^2 \theta - \tan \theta = 5$, $3 \cos \theta + 2 \sin \theta = 1$.				
Differentiation	use the derivatives of e^x , $\ln x$, $\sin x$, $\cos x$, $\tan x$, together with constant multiples, sums, differences and composites	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	differentiate products and quotients e.g. $\frac{2x-4}{3x+2}$, $x^2 \ln x$, xe^{1-x^2}	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	find and use the first derivative of a function which is defined parametrically or implicitly. e.g. $x = t - e^{2t}$, $y = t + e^{2t}$. e.g. $x^2 + y^2 = xy + 7$. Including use in problems involving tangents and normals.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
Integration	extend the idea of 'reverse differentiation' to include the integration of e^{ax+b} , $\frac{1}{ax+b}$, $\sin(ax+b)$, $\cos(ax+b)$ and $\sec^2(ax+b)$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	Knowledge of the general method of integration by substitution is not required				
	use trigonometrical relationships in carrying out integration e.g. use of double-angle formulae to integrate \sin^2 or $\cos^2(2x)$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	

Topic	You should be able to	R	A	G	Comments
	understand and use the trapezium rule to estimate the value of a definite integral Includes use of sketch graphs in simple cases to determine whether the trapezium rule gives an over-estimate or an under-estimate	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
Numerical solution of equations	locate approximately a root of an equation, by means of graphical considerations and/or searching for a sign change e.g. finding a pair of consecutive integers between which a root lies	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	understand the idea of, and use the notation for, a sequence of approximations which converges to a root of an equation	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	understand how a given simple iterative formula of the form $x_{n+1} = F(x_n)$ relates to the equation being solved, and use a given iteration, or an iteration based on a given rearrangement of an equation, to determine a root to a prescribed degree of accuracy	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	Knowledge of the condition for convergence is not included, but an understanding that an iteration may fail to converge is expected				

Paper 3 Pure Mathematics

Topic	You should be able to	R	A	G	Comments
Algebra	understand the meaning of $ x $, sketch the graph of $y = ax + b $ and use relations such as $ a = b \Leftrightarrow a^2 = b^2$ and $ x - a < b \Leftrightarrow a - b < x < a + b$ when solving equations and inequalities Graphs of $y = f(x) $ and $y = f(x)$ for non-linear functions f are not included. e.g. $ 3x - 2 = 2x + 7 $, $2x + 5 < x + 1 $.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	divide a polynomial, of degree not exceeding 4, by a linear or quadratic polynomial, and identify the quotient and remainder (which may be zero)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	use the factor theorem and the remainder theorem e.g. to find factors and remainders, solve polynomial equations or evaluate unknown coefficients. Factors of the form $(ax + b)$ in which the coefficient of x is not unity are included, and similarly for calculation of remainders	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	

Topic	You should be able to	R	A	G	Comments
	recall an appropriate form for expressing rational functions in partial fractions, and carry out the decomposition, in cases where the denominator is no more complicated than <ul style="list-style-type: none"> • $(ax + b)(cx + d)(ex + f)$ • $(ax + b)(cx + d)^2$ • $(ax + b)(cx^2 + d)$ Excluding cases where the degree of the numerator exceeds that of the denominator	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	use the expansion of $(1 + x)^n$, where n is a rational number and $ x < 1$. Finding the general term in an expansion is not included. Adapting the standard series to expand e.g. $(2 - \frac{1}{2}x)^{-1}$ is included, and determining the set of values of x for which the expansion is valid in such cases is also included.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
Logarithmic and exponential functions	understand the relationship between logarithms and indices, and use the laws of logarithms (excluding change of base)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	understand the definition and properties of e^x and $\ln x$, including their relationship as inverse functions and their graphs	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	Knowledge of the graph of $y = e^{kx}$ for both positive and negative values of k is included.				
	use logarithms to solve equations and inequalities in which the unknown appears in indices	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	e.g. $2^x < 5$, $3 \times 2^{3x-1} < 5$, $3^{x+1} = 4^{2x-1}$				
	use logarithms to transform a given relationship to linear form, and hence determine unknown constants by considering the gradient and/or intercept	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	e.g. $y = kx^n$, $y = k(a^x)$				

Topic	You should be able to	R	A	G	Comments
Trigonometry	properties and graphs of all six trigonometric functions for angles of any magnitude	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	use trigonometrical identities for the simplification and exact evaluation of expressions and in the course of solving equations, and select an identity or identities appropriate to the context, showing familiarity in particular with the use of <ul style="list-style-type: none"> • $\sec^2 \theta \equiv 1 + \tan^2 \theta$ and $\operatorname{cosec}^2 \theta \equiv 1 + \cot^2 \theta$, • the expansions of $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$ • the formulae for $\sin 2A$, $\cos 2A$ and $\tan 2A$ • the expression of $a \sin \theta + b \cos \theta$ in the forms $R \sin(\theta \pm \alpha)$ and $R \cos(\theta \pm \alpha)$. e.g. simplifying $\cos(x - 30^\circ) - 3 \sin(x - 60^\circ)$. e.g. solving $\tan \theta + \cot \theta = 4$, $2 \sec^2 \theta - \tan \theta = 5$, $3 \cos \theta + 2 \sin \theta = 1$.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
Differentiation	use the derivatives of e^x , $\ln x$, $\sin x$, $\cos x$, $\tan x$, $\tan^{-1} x$, and together with constant multiples, sums, differences and composites Derivatives of $\sin^{-1} x$ and $\cos^{-1} x$ are not required. differentiate products and quotients e.g. $\frac{2x - 4}{3x + 2}$, $x^2 \ln x$, $x e^{1-x^2}$ find and use the first derivative of a function which is defined parametrically or implicitly e.g. $x = t - e^{2t}$, $y = t + e^{2t}$. e.g. $x^2 + y^2 = xy + 7$. Including use in problems involving tangents and normals.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
Integration	extend the idea of 'reverse differentiation' to include the integration of e^{ax+b} , $\frac{1}{ax+b}$, $\sin(ax+b)$, $\cos(ax+b)$, $\sec^2(ax+b)$ and $\frac{1}{x^2+a^2}$ Including examples such as $\frac{1}{2+3x^2}$ use trigonometrical relationships in carrying out integration e.g. use of double-angle formulae to integrate $\sin^2 x$ or $\cos^2(2x)$. integrate rational functions by means of decomposition into partial fractions Restricted to types of partial fractions as specified in section 1 above	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	

Topic	You should be able to	R	A	G	Comments
	recognise an integrand of the form $\frac{kf'(x)}{f(x)}$, and integrate such functions e.g. integration of $\frac{x}{x^2+1}$, $\tan x$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	recognise when an integrand can usefully be regarded as a product, and use integration by parts e.g. integration of $x \sin 2x$, x^2e^{-x} , $\ln x$, $x \tan^{-1} x$.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	use a given substitution to simplify and evaluate either a definite or an indefinite integral e.g. to integrate $\sin^2 2x \cos x$ using the substitution $u = \sin x$.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
Numerical solution of equations	locate approximately a root of an equation, by means of graphical considerations and/or searching for a sign change e.g. finding a pair of consecutive integers between which a root lies	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	understand the idea of, and use the notation for, a sequence of approximations which converges to a root of an equation	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	understand how a given simple iterative formula of the form $x_{n+1} = F(x_n)$ relates to the equation being solved, and use a given iteration, or an iteration based on a given rearrangement of an equation, to determine a root to a prescribed degree of accuracy Knowledge of the condition for convergence is not included, but an understanding that an iteration may fail to converge is expected	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
Vectors	use standard notations for vectors, i.e. $\begin{pmatrix} x \\ y \end{pmatrix}$, $x\mathbf{i} + y\mathbf{j}$, $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$, $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, \vec{AB} , \mathbf{a}	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	carry out addition and subtraction of vectors and multiplication of a vector by a scalar, and interpret these operations in geometrical terms e.g. 'OABC is a parallelogram' is equivalent to $\vec{OB} = \vec{OA} + \vec{OC}$. The general form of the ratio theorem is not included, but understanding that the midpoint of AB has position vector $\frac{1}{2}(\vec{OA} + \vec{OB})$ is expected.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	calculate the magnitude of a vector, and use unit vectors, displacement vectors and position vectors In 2 or 3 dimensions	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	

Topic	You should be able to	R	A	G	Comments
	understand the significance of all the symbols used when the equation of a straight line is expressed in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$, and find the equation of a line, given sufficient information	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	e.g. finding the equation of a line given the position vector of a point on the line and a direction vector, or the position vectors of two points on the line				
	determine whether two lines are parallel, intersect or are skew, and find the point of intersection of two lines when it exists	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	Calculation of the shortest distance between two skew lines is excluded. Finding the equation of the common perpendicular to two skew lines is also excluded				
	calculate the scalar product of two vectors, and use scalar products in problems involving lines and points e.g. finding the angle between two lines, & finding the foot of the perpendicular from a point to a line; questions may involve 3D objects such as cuboids, tetrahedra (pyramids), etc. Knowledge of the vector product is not required	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
Differential equations	formulate a simple statement involving a rate of change as a differential equation The introduction and evaluation of a constant of proportionality, where necessary, is included	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	find by integration a general form of solution for a first order differential equation in which the variables are separable Any of the integration techniques from section 5 above may be required	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	use an initial condition to find a particular solution	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	interpret the solution of a differential equation in the context of a problem being modelled by the equation	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	Where a differential equation is used to model a 'real-life' situation, no specialised knowledge of the context will be required				

Topic	You should be able to	R	A	G	Comments
Complex numbers	understand the idea of a complex number, recall the meaning of the terms real part, imaginary part, modulus, argument, conjugate, and use the fact that two complex numbers are equal if and only if both real and imaginary parts are equal	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	Notations $\operatorname{Re} z$, $\operatorname{Im} z$, $ z $, $\arg z$, z^* should be known. (The argument of a complex number will usually refer to an angle θ such that $-\pi < \theta \leq \pi$, but in some cases the interval $0 \leq \theta < 2\pi$ may be more convenient. Answers may use either interval unless the question specifies otherwise.)				
	carry out operations of addition, subtraction, multiplication and division of two complex numbers expressed in Cartesian form $x + iy$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	For calculations involving multiplication or division, full details of the working should be shown				
	use the result that, for a polynomial equation with real coefficients, any non-real roots occur in conjugate pairs	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	e.g. in solving a cubic or quartic equation where one complex root is given				
	represent complex numbers geometrically by means of an Argand diagram	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	carry out operations of multiplication and division of two complex numbers expressed in polar form $r(\cos \theta + i \sin \theta) \equiv re^{i\theta}$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	The results $ z_1 z_2 = z_1 z_2 $ and $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$, and corresponding results for division, are included				
	find the two square roots of a complex number	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
e.g. the square roots of $5 + 12i$ in exact Cartesian form. Full details of the working should be shown					
understand in simple terms the geometrical effects of conjugating a complex number and of adding, subtracting, multiplying and dividing two complex numbers	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>		
illustrate simple equations and inequalities involving complex numbers by means of loci in an Argand diagram	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>		
e.g. $ z - a < k$, $ z - a = z - b $, $\arg(z - a) = \alpha$.					

Paper 4 Mechanics

Topic	You should be able to	R	A	G	Comments	
Forces and equilibrium	identify the forces acting in a given situation e.g. by drawing a force diagram	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>		
	understand the vector nature of force, and find and use components and resultants Calculations are always required, not approximate solutions by scale drawing	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>		
	use the principle that, when a particle is in equilibrium, the vector sum of the forces acting is zero, or equivalently, that the sum of the components in any direction is zero Solutions by resolving are usually expected, but equivalent methods (e.g. triangle of forces, Lami's Theorem, where suitable) are entirely acceptable; any such additional methods are not required knowledge, however, and will not be referred to in questions	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>		
	understand that a contact force between two surfaces can be represented by two components, the normal component and the frictional component	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>		
	use the model of a 'smooth' contact, and understand the limitations of this model	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>		
	understand the concepts of limiting friction and limiting equilibrium, recall the definition of coefficient of friction, and use the relationship $F = \mu R$ or $F \leq \mu R$, as appropriate Terminology such as 'about to slip' may be used to mean 'in limiting equilibrium' in questions	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>		
	use Newton's third law e.g. the force exerted by a particle on the ground is equal and opposite to the force exerted by the ground on the particle	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>		
	Kinematics of motion in a straight line	understand the concepts of distance and speed as scalar quantities, and of displacement, velocity and acceleration as vector quantities Restricted to motion in one dimension only. The term 'deceleration' may sometimes be used in the context of decreasing speed	<input type="checkbox"/>	<input type="checkbox"/>		<input type="checkbox"/>

Topic	You should be able to	R	A	G	Comments
	sketch and interpret displacement–time graphs and velocity–time graphs, and in particular appreciate that <ul style="list-style-type: none"> the area under a velocity–time graph represents displacement, the gradient of a displacement–time graph represents velocity, the gradient of a velocity–time graph represents acceleration 	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	use differentiation and integration with respect to time to solve simple problems concerning displacement, velocity and acceleration Calculus required is restricted to that within the content list for component Pure Mathematics 1	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	use appropriate formulae for motion with constant acceleration in a straight line Problems may involve setting up more than one equation, using information about the motion of different particles	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
Momentum	use the definition of linear momentum and show understanding of its vector nature For motion in one dimension only	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	use conservation of linear momentum to solve problems that may be modelled as the direct impact of two bodies The 'direct impact' of two bodies includes the case where the bodies coalesce on impact. Knowledge of impulse and the coefficient of restitution is not required	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
Newton's laws of motion	apply Newton's laws of motion to the linear motion of a particle of constant mass moving under the action of constant forces, which may include friction, tension in an inextensible string and thrust in a connecting rod If any other forces resisting motion are to be considered (e.g. air resistance) this will be indicated in the question	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	use the relationship between mass and weight $W = mg$. In this component, questions are mainly numerical, and use of the approximate numerical value $10 \text{ (m s}^{-2}\text{)}$ for g is expected in such cases	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	solve simple problems which may be modelled as the motion of a particle moving vertically or on an inclined plane with constant acceleration Including, for example, motion of a particle on a rough plane where the acceleration while moving up the plane is different from the acceleration while moving down the plane	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	

Topic	You should be able to	R	A	G	Comments
	solve simple problems which may be modelled as the motion of connected particles e.g. particles connected by a light inextensible string passing over a smooth pulley, or a car towing a trailer by means of either a light rope or a light rigid tow-bar	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
Energy, work and power	understand the concept of the work done by a force, and calculate the work done by a constant force when its point of application undergoes a displacement not necessarily parallel to the force $W = Fd \cos \theta$; use of the scalar product is not required.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	understand the concepts of gravitational potential energy and kinetic energy, and use appropriate formulae	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	understand and use the relationship between the change in energy of a system and the work done by the external forces, and use in appropriate cases the principle of conservation of energy Including cases where the motion may not be linear (e.g. a child on a smooth curved 'slide') so long as only overall energy changes need to be considered	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	use the definition of power as the rate at which a force does work, and use the relationship between power, force and velocity for a force acting in the direction of motion Including calculation of (average) power as $\frac{\text{Work done}}{\text{Time taken}}$. $P = Fv$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	solve problems involving, for example, the instantaneous acceleration of a car moving on a hill against a resistance	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	

Paper 5 Probability & Statistics 1

Topic	You should be able to	R	A	G	Comments
	select a suitable way of presenting raw statistical data, and discuss advantages and/or disadvantages that particular representations may have	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	construct and interpret stem-and-leaf diagrams, box-and-whisker plots, histograms and cumulative frequency graphs Including back-to-back stem-and-leaf diagrams	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	understand and use different measures of central tendency (mean, median, mode) and variation (range, interquartile range, standard deviation) e.g. in comparing and contrasting sets of data	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	

Topic	You should be able to	R	A	G	Comments
	calculate and use the mean and standard deviation of a set of data (including grouped data) either from the data itself or from given totals Σx and Σx^2 , or coded totals $\Sigma(x - a)$ and $\Sigma(x - a)^2$, and use such totals in solving problems which may involve up to two data sets.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
Permutations and combinations	understand the terms permutation and combination, and solve simple problems involving selections	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	solve problems about arrangements of objects in a line, including those involving <ul style="list-style-type: none"> repetition (e.g. the number of ways of arranging the letters of the word 'NEEDLESS'), restriction (e.g. the number of ways several people can stand in a line if two particular people must — or must not — stand next to each other) 	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	Problems may include cases such as people sitting in two (or more) rows. Problems about objects arranged in a circle are excluded				
Probability	evaluate probabilities in simple cases by means of enumeration of equiprobable elementary events, or by calculation using permutations or combinations e.g. the total score when two fair dice are thrown. e.g. drawing balls at random from a bag containing balls of different colours	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	use addition and multiplication of probabilities, as appropriate, in simple cases Explicit use of the general formula $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ is not required.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	understand the meaning of exclusive and independent events, including determination of whether events A and B are independent by comparing the values of $P(A \cap B)$ and $P(A) \times P(B)$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	calculate and use conditional probabilities in simple cases. e.g. situations that can be represented by a sample space of equiprobable elementary events, or a tree diagram. The use of $P(A B) = \frac{P(A \cap B)}{P(B)}$ may be required in simple cases	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	construct a probability distribution table relating to a given situation involving a discrete random variable X , and calculate $E(X)$ and $\text{Var}(X)$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	use formulae for probabilities for the binomial and geometric distributions, and recognise practical situations where these distributions are suitable models The notations $B(n, p)$ and $\text{Geo}(p)$ are included. $\text{Geo}(p)$ denotes the distribution in which $p_r = p(1 - p)^{r-1}$ for $r = 1, 2, 3, \dots$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
Discrete random variables	use formulae for probabilities for the binomial and geometric distributions, and recognise practical situations where these distributions are suitable models The notations $B(n, p)$ and $\text{Geo}(p)$ are included. $\text{Geo}(p)$ denotes the distribution in which $p_r = p(1 - p)^{r-1}$ for $r = 1, 2, 3, \dots$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	

Topic	You should be able to	R	A	G	Comments
	use formulae for the expectation and variance of the binomial distribution and for the expectation of the geometric distribution Proofs of formulae are not required	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
The normal distribution	understand the use of a normal distribution to model a continuous random variable, and use normal distribution tables Sketches of normal curves to illustrate distributions or probabilities may be required	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	solve problems concerning a variable X , where $X \sim N(\mu, \sigma^2)$, including <ul style="list-style-type: none"> finding the value of $P(X > x_1)$, or a related probability, given the values of x_1, μ, σ. finding a relationship between x_1, μ and σ given the value of $P(X > x_1)$ or a related probability 	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	recall conditions under which the normal distribution can be used as an approximation to the binomial distribution, and use this approximation, with a continuity correction, in solving problems n sufficiently large to ensure that both $np > 5$ and $nq > 5$.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	

Paper 6 Probability & Statistics 2

Topic	You should be able to	R	A	G	Comments
The Poisson distribution	calculate probabilities for the distribution $Po(\lambda)$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	use the fact that if $X \sim Po(\lambda)$ then the mean and variance of X are each equal to λ . Proofs are not required	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	understand the relevance of the Poisson distribution to the distribution of random events, and use the Poisson distribution as a model	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	use the Poisson distribution as an approximation to the binomial distribution where appropriate The conditions that n is large and p is small should be known; $n > 50$ and $np < 5$, approximately	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	use the normal distribution, with continuity correction, as an approximation to the Poisson distribution where appropriate The condition that λ is large should be known; $\lambda > 15$, approximately	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	

Topic	You should be able to	R	A	G	Comments
Linear combinations of random variables	use, in the course of solving problems, the results that: <ul style="list-style-type: none"> $E(aX + b) = aE(X) + b$ and $\text{Var}(aX + b) = a^2 \text{Var}(X)$ $E(aX + bY) = aE(X) + bE(Y)$ $\text{Var}(aX + bY) = a^2 \text{Var}(X) + b^2 \text{Var}(Y)$ for independent X and Y if X has a normal distribution then so does $aX + b$ if X and Y have independent normal distributions then $aX + bY$ has a normal distribution if X and Y have independent Poisson distributions then $X + Y$ has a Poisson distribution. 	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	Proofs of these results are not required				
Continuous random variables	understand the concept of a continuous random variable, and recall and use properties of a probability density function	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	For density functions defined over a single interval only; the domain may be infinite, e.g. $\frac{3}{x^4}$ for $x \geq 1$				
	use a probability density function to solve problems involving probabilities, and to calculate the mean and variance of a distribution	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	Location of the median or other percentiles of a distribution by direct consideration of an area using the density function may be required but explicit knowledge of the cumulative distribution function is not included				
Sampling and estimation	understand the distinction between a sample and a population, and appreciate the necessity for randomness in choosing samples	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	explain in simple terms why a given sampling method may be unsatisfactory	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	Knowledge of particular sampling methods, such as quota or stratified sampling, is not required, but an elementary understanding of the use of random numbers in producing random samples is included				
	recognise that a sample mean can be regarded as a random variable, and use the facts that $E(\bar{X}) = \mu$ and that $\text{Var}(\bar{X}) = \frac{\sigma^2}{n}$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	use the fact that \bar{X} has a normal distribution if X has a normal distribution	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	use the Central Limit Theorem where appropriate	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	Only an informal understanding of the Central Limit Theorem (CLT) is required; for large sample sizes, the distribution of a sample mean is approximately normal				

Topic	You should be able to	R	A	G	Comments
	calculate unbiased estimates of the population mean and variance from a sample, using either raw or summarised data	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	Only a simple understanding of the term 'unbiased' is required, e.g. that although individual estimates will vary the process gives an accurate result 'on average'				
	determine and interpret a confidence interval for a population mean in cases where the population is normally distributed with known variance or where a large sample is used	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	determine, from a large sample, an approximate confidence interval for a population proportion				
Hypothesis tests	understand the nature of a hypothesis test, the difference between one-tailed and two-tailed tests, and the terms null hypothesis, alternative hypothesis, significance level, rejection region (or critical region), acceptance region and test statistic	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	Outcomes of hypothesis tests are expected to be interpreted in terms of the contexts in which questions are set				
	formulate hypotheses and carry out a hypothesis test in the context of a single observation from a population which has a binomial or Poisson distribution, using <ul style="list-style-type: none"> • direct evaluation of probabilities, • a normal approximation to the binomial or the Poisson distribution, where appropriate 	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	formulate hypotheses and carry out a hypothesis test concerning the population mean in cases where the population is normally distributed with known variance or where a large sample is used	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	understand the terms Type I error and Type II error in relation to hypothesis tests	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	
	calculate the probabilities of making Type I and Type II errors in specific situations involving tests based on a normal distribution or direct evaluation of binomial or Poisson probabilities	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	

Section 6: Useful websites

The websites listed below are useful resources to help you study for your Cambridge International AS & A Level Mathematics.

www.s-cool.co.uk/a-level/maths

Good coverage of Pure Mathematics and Statistics topics, but not Mechanics. Revision material is arranged by topic and includes explanations, revision summaries and exam-style questions with answers.

<http://www.cimt.org.uk/>

This site contains useful course materials – textbook style notes, worked examples and exercises – on Pure Mathematics, Mechanics and Probability & Statistics topics. Look for the MEP link to access these. Answers to the exercises are available to teachers who can request a password.

www.khanacademy.org

A site with many useful video explanations and some exercises. Uses American terminology but still helpful for revision purposes. You can search by topic or by school grade (year).

www.mathcentre.ac.uk

Notes, examples and exercises for many of the topics on the syllabus.

www.physicsandmathstutor.com

A source of useful revision notes, exercises and sets of questions from UK examination papers that have been organised by difficulty.

<https://nrich.maths.org/9088>

Resources for learners aged 16+ from NRICH at Cambridge University.

<https://mrbartonmaths.com/students/a-level>

Notes, videos and examples on various AS and A Level topics. Also includes exam papers and mark schemes for UK exam boards.

www.getrevising.co.uk/timetable

A study planner you can use to help you plan your revision.

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